MDM4U **Organized Counting with Venn Diagrams** **Date: _________**

A **set** is a collection of distinct objects. We often notate sets with capital letters.

e.g. $A = \{x \mid x \geq 5\}$ $B = \{a, b, c, d\}$ $C = \emptyset = \{\ \}$ or empty set $D = \{0\}$

The objects *a*,*b*,*c*,*d* are called ________________or ______________ of the set B.

Note: Set C does not have any element and is called **empty set** or **null set.**

Does set D represent a null set?

To denote the number of elements in a set A, we write **n(A).**

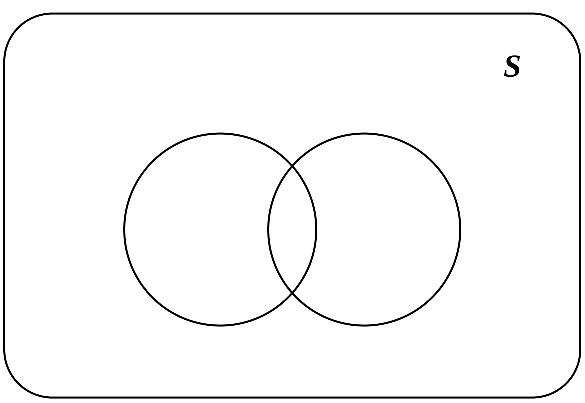
Relationships between sets and their subsets can be illustrated using **Venn diagrams** in which sets are represented by shaded or coloured geometrical shapes.

Intersection of Sets

Given two sets, ***A*** and ***B***, if ***A*** and ***B*** overlap or have elements in common, the set of common elements is called the intersection of ***A*** and ***B***, and is written as $A \cap B$.

These common elements are members of set ***A*** and are also elements of set ***B***. Consequently, $A \cap B =$ {elements in both ***A*** **AND** ***B***}

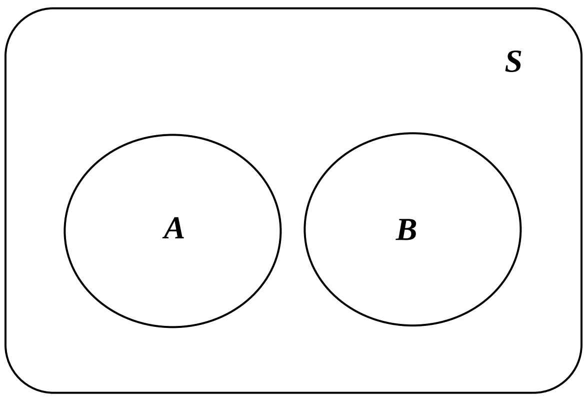
The set $A \cap B$ is represented by the region of overlap of the two sets in the Venn diagram below. Sets ***A*** and ***B*** exist as sets within the larger set ***S***, called the **universal set.** They are **subsets** of the set ***S***.



Disjoint Sets

If ***A*** and ***B*** have no elements in common (i.e., $n(A \cap B) = 0$), they are said to be disjoint and their intersection is the empty set, represented by the Greek letter $\varnothing$.
(i.e., $A \cap B = \varnothing$).

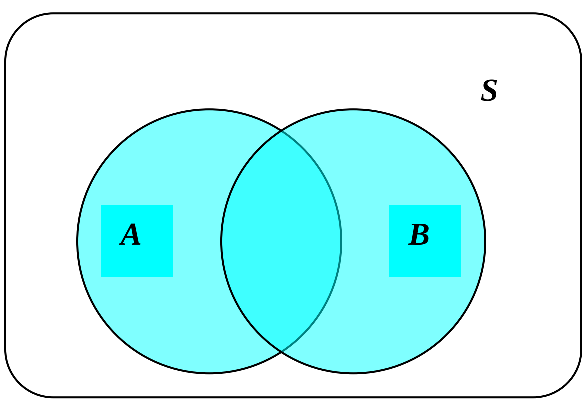
The diagram below shows disjoint sets A and B.



Union of Sets

The set formed by combining the elements of ***A*** with those in ***B*** is called the union of ***A*** and ***B***, and is written as $A \cup B$.
The elements in $A \cup B$ are elements of ***A*** or they are elements of ***B***. Consequently,
$A \cup B = \{$elements in ***A* OR *B***$\}$

The set $A \cup B$ is represented by the shaded area in the diagram below.



Example 1: Using a Venn Diagram to Solve a Counting Problem

Suppose a survey of 100 Grade 12 mathematics students in a local high school produced the following results.

Math Course Taken	Number of Students
Advanced Functions and Introductory Calculus	80
Geometry and Discrete Math	33
Data Management	68
Geometry and Discrete Math and Calculus	30
Geometry and Discrete Math and Data Management	6
Data Management and Calculus	50
All three courses	5

How many students are enrolled in Calculus and in no other mathematics course?

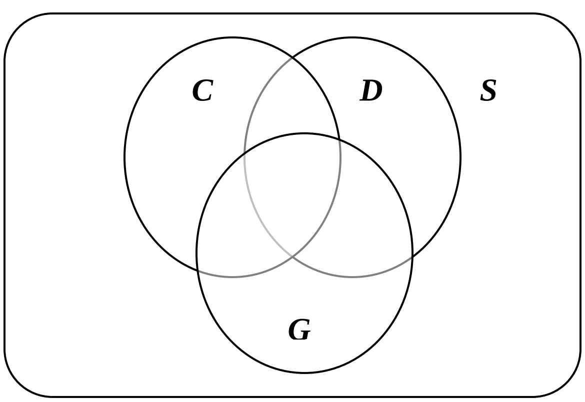
How many students are enrolled in Calculus? Or Data Management?

First construct a Venn diagram to represent the three groups of students. For convenience, we can label the sets as follow:
C for Calculus
G for Geometry and Discrete Math
D for Data Management.

The entire sample space, S, will consist of all the students in Grade 12.

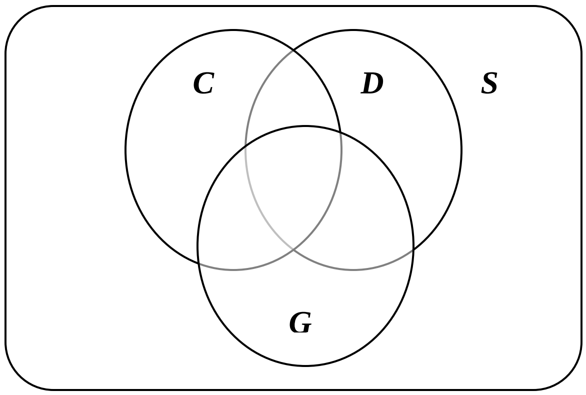
To avoid double counting, start entering information from the chart in the very middle of the diagram and work your way out.

$n(C \cap G \cap D) = 5$



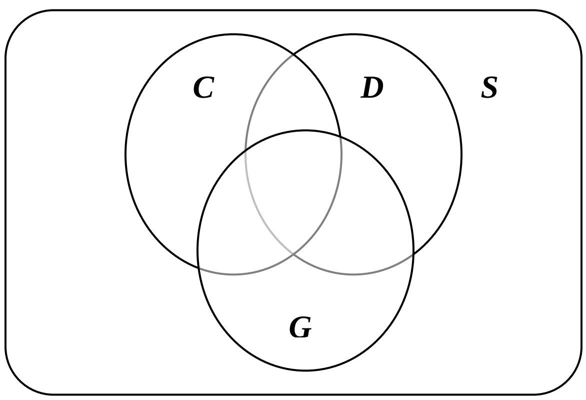
Now since $n(G \cap C) = 30$ and $n(C \cap G \cap D) = 5$

The number of students who take Calculus and Geometry but not Data Management must be ______



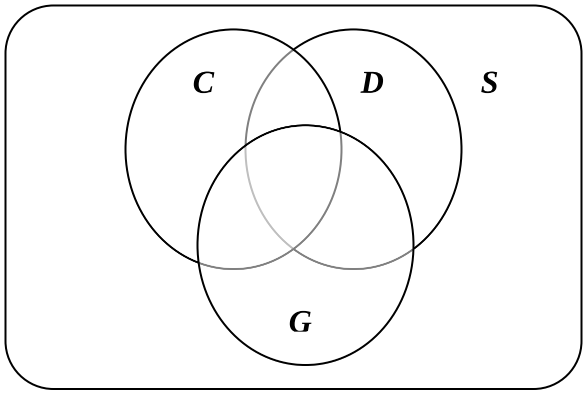
Since $n(D \cap C) = 50$ and $n(C \cap G \cap D) = 5$

The number of students who take Calculus and Data but not Geometry must be ______.



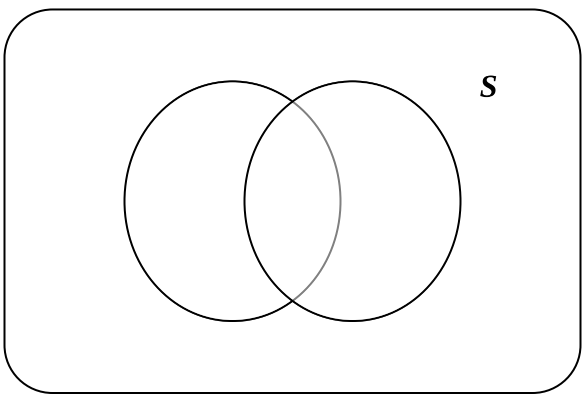
There are 80 students enrolled in Calculus, of which ________ have been accounted for. There must be ______ students who take only Calculus and no other mathematics course.

The complete Venn diagram:



If we consider only the students taking Calculus and Data Management, the Venn Diagram will have only two sets and one intersection. The total number of students in both courses is ________.

$n(C \cup D) = n(C) + n(D) - n(C \cap D) =$ ________ + _________ - ________ = _______



Additive Principle for Unions of Two Sets

Given two sets, A and B, the number of elements in $A \cup B$ can be found by totalling the number of elements in both sets and then subtracting the number that have been counted twice. The double-counted elements will be found in the intersection of the two sets.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$