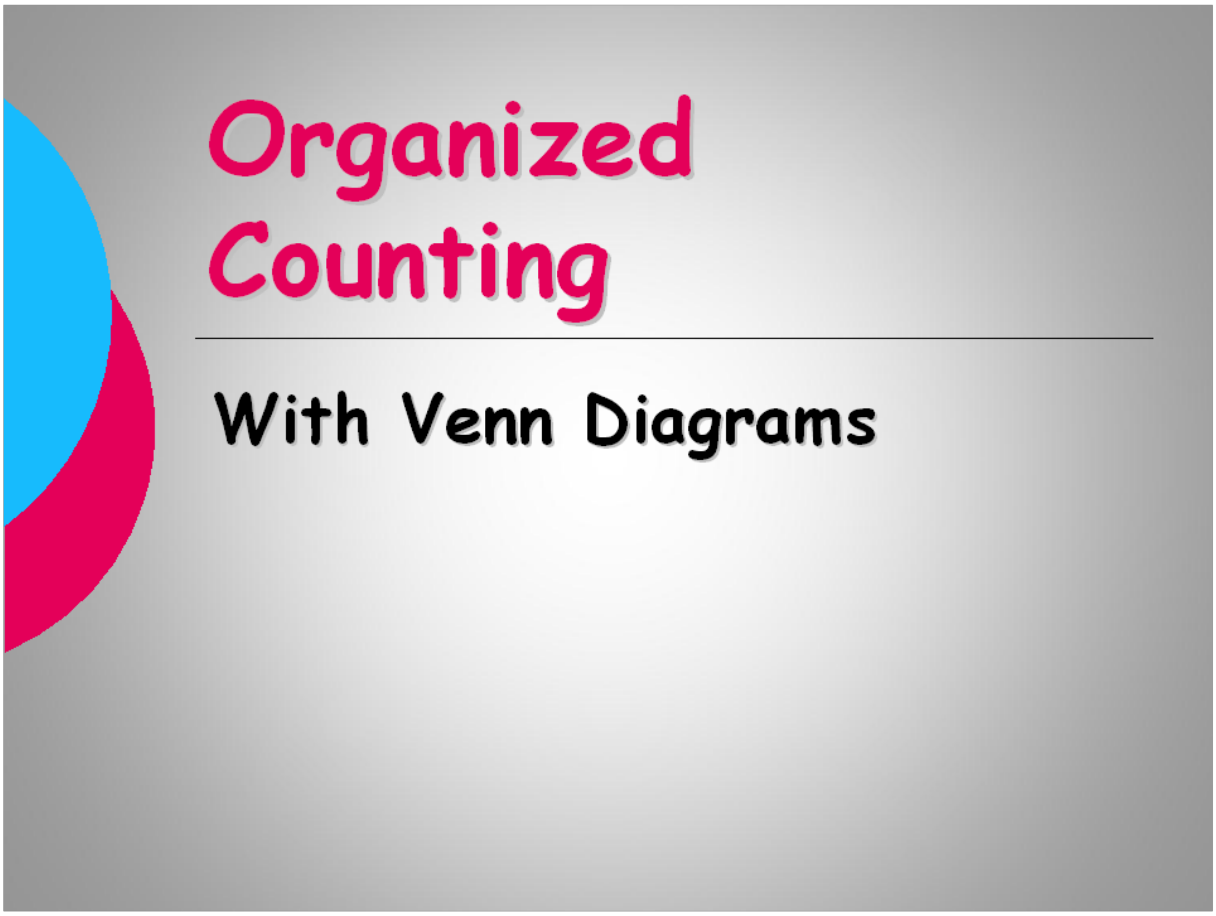
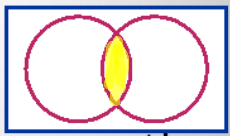

Organized
Counting
With Venn Diagrams

Definitions

- **Venn Diagram**
  - A useful way of representing various sets.
- **Set**
  - A collection of distinguishable or different elements.
  - Denoted by circles in a Venn diagram.
- **Elements**
  - The individual members of a set.

Definitions (continued)

- **Universal Set**
  - The set S, from which all the elements are derived.
  - Denoted by a rectangular box enclosing the sets.



- **Common Elements**
  - Elements which belong to more than one set.
  - Illustrated by using overlapping circles.

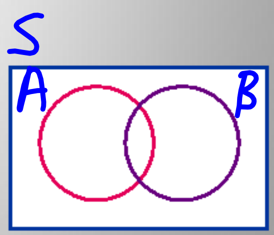
Definitions (continued)

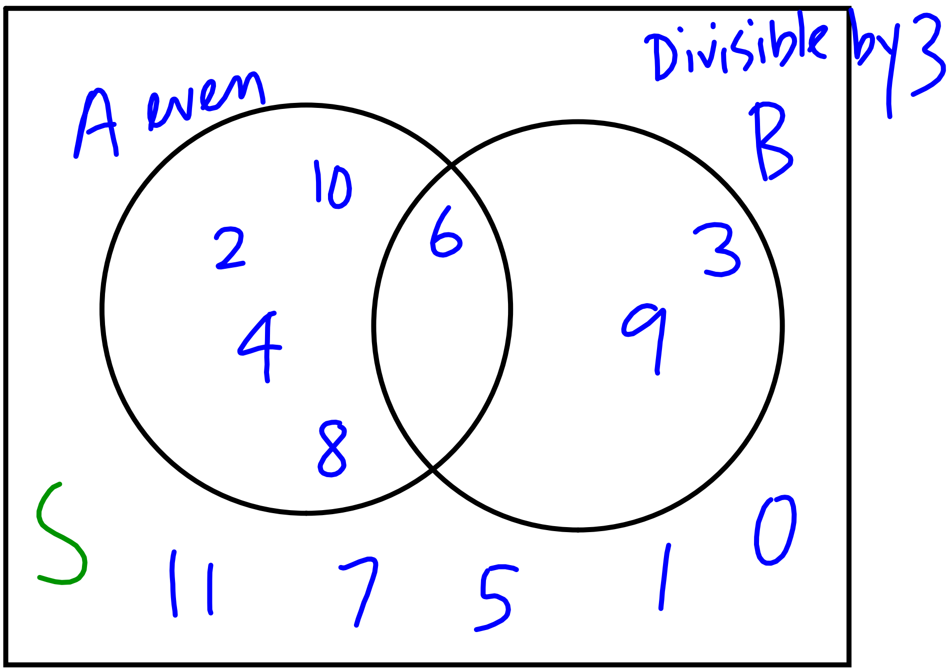
- Common Elements (continued)
  - Example: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

    If S = the set of whole numbers less than 12

    A = the set of even numbers less than 12

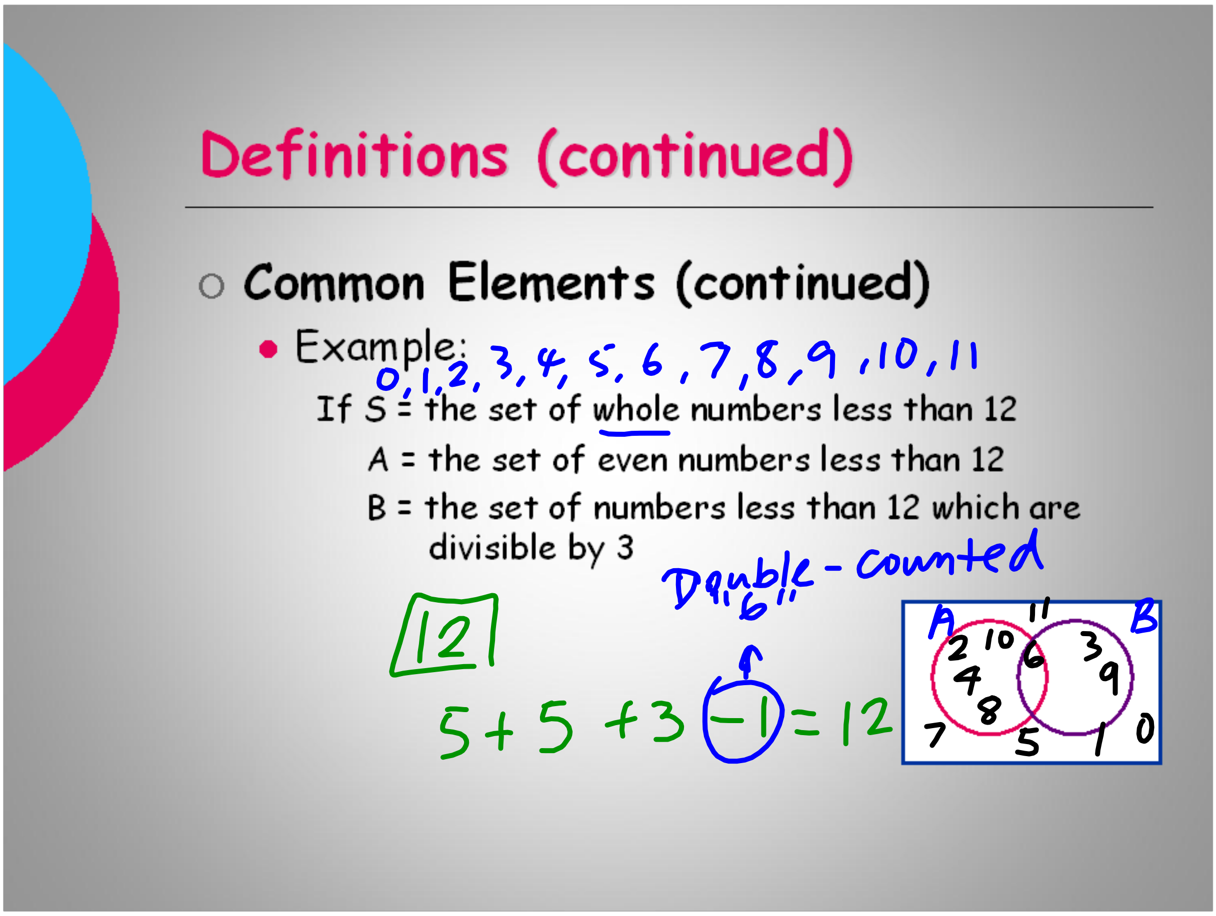
    B = the set of numbers less than 12 which are divisible by 3





$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 5 + 3 - 1$$
$$= 7$$

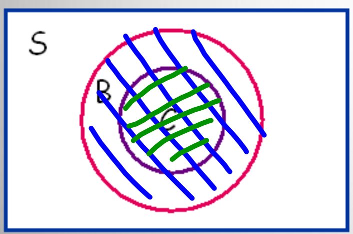

Definitions (continued)
Common Elements (continued)
Example:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
If S = the set of whole numbers less than 12
A = the set of even numbers less than 12
B = the set of numbers less than 12 which are divisible by 3
Double-counted
"6"
12
5 + 5 + 3 − 1 = 12
A
B
2 10 4 8
6
3 9
11 7 5 1 0

Definitions (continued)

- **Subset of a Set**
  - "A" is a subset of "B" if all the elements in A are also in B.
  - Example: If S = { real numbers }

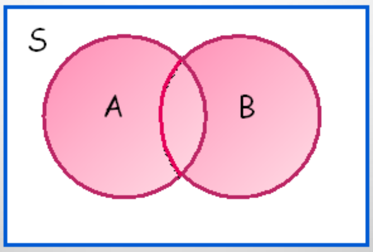
    B = { whole numbers }

    C = { even whole numbers }



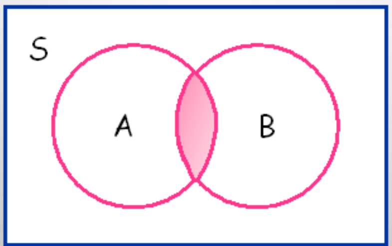
Key Ideas

- Union of Sets
  - The union of sets A and B is denoted by $A \cup B$. "OR"
  - It consists of all elements which are A or B.



Key Ideas (continued)

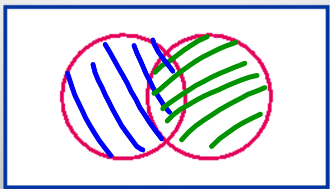
- **Intersection of Sets**
  - The intersection of sets A and B is denoted by <u>A ∩ B</u>. "AND"
  - It consists of all elements which are in A <u>and</u> B.



Key Ideas (continued)

- Principle of Inclusion and Exclusion
  - For sets A and B, the total number of elements in either A or B is the number in A plus the number in B minus the number in both A and B.

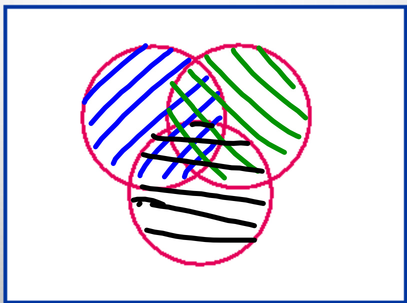
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



Key Ideas (continued)

- **Principle of Inclusion and Exclusion (cont)**
  - It can also be applied to three or more sets.

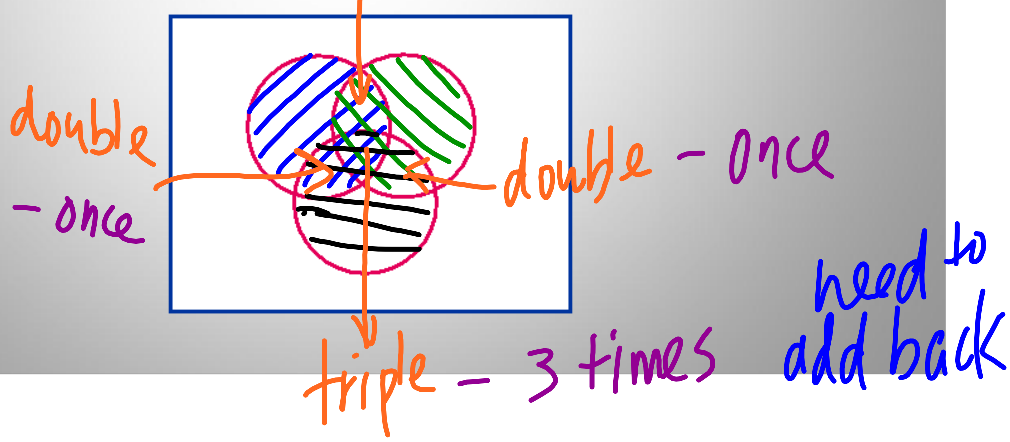
  $n(A \cup B \cup C) =$



Key Ideas (continued)

- Principle of Inclusion and Exclusion (cont)
  - It can also be applied to three or more sets.

$n(A \cup B \cup C) =$ double – once



Draw the Venn Diagram

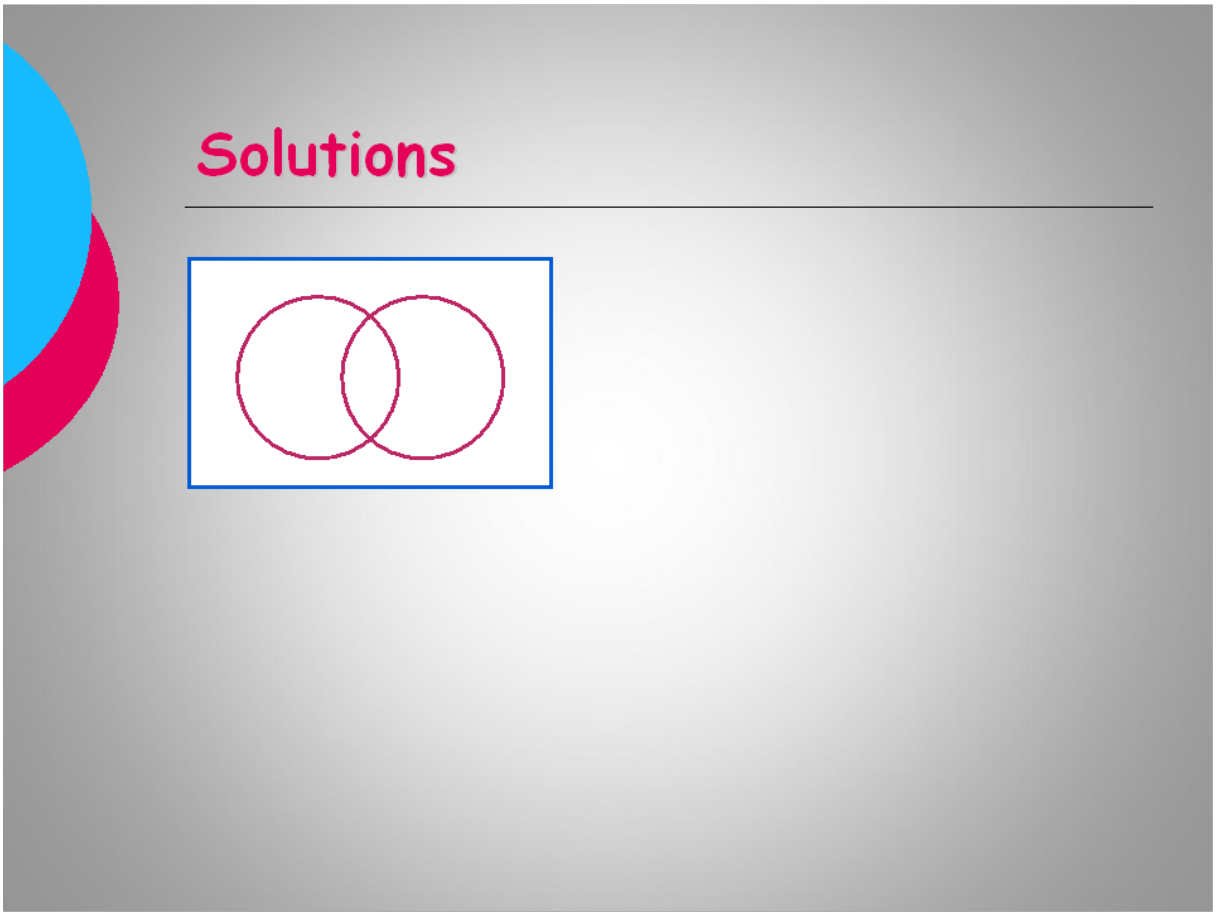
- Given two sets, A and B, the number of elements in $A \cup B$ can be found by totalling the number of elements in both sets and then subtracting the number that have been counted twice.
- The double-counted elements will be found in the intersection of the two sets.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Examples

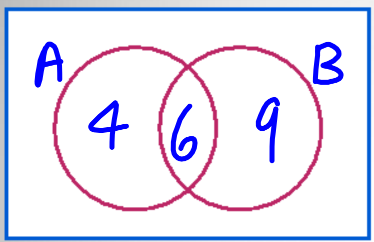
1. There are 10 students on the volleyball team and 15 on the basketball team. When planning a field trip for both teams, the coach arranges transportation for 19 students.

   a) Why?
   b) Draw a Venn Diagram
   c) How many students are on both teams?
   d) What might the set S represent?


Solutions

volleyball Basketball

Solutions $n(A \cap B)$ $n(A)$ $n(B)$ $n(A \cup B)$



$n(A) = 10$

$n(B) = 15$

$n(A \cup B) = 19$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

* $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 10 + 15 - 19$

$= 6$

Examples

1. There are 10 students on the volleyball team and 15 on the basketball team. When planning a field trip for both teams, the coach arranges transportation for 19 students.

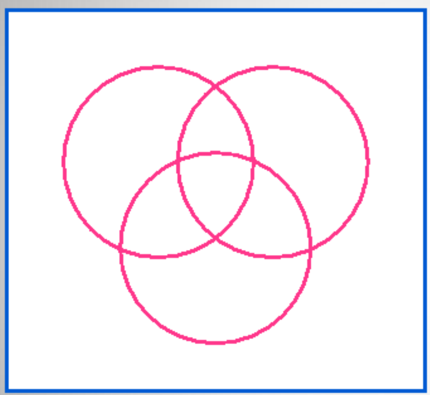
   a) Why?
   b) Draw a Venn Diagram
   c) How many students are on both teams? 6
   d) What might the set S represent?

   All the students of the school

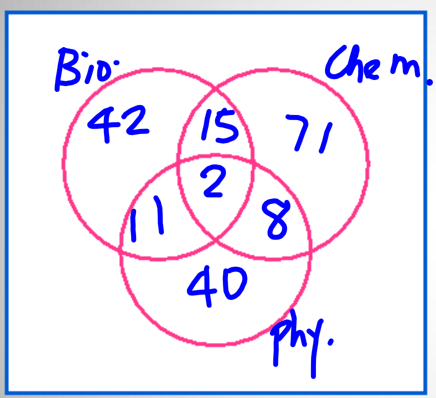
Examples (continued)

2. There are 140 students in Grade 12. Of them, 42 take Biology, 71 take Chemistry, 40 take Physics, 15 take Biology and Chemistry, 8 take Chemistry and Physics, 11 take Biology and Physics, and 2 take all three sciences. How many do not take any sciences at all?

Solutions (continued)



Solutions (continued)



$A \cup B \cup C = 121$

$S = 140$

Not taking science

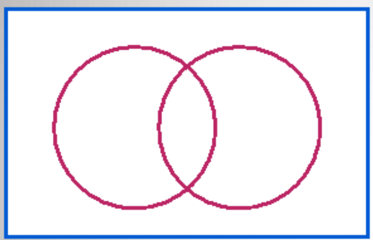
$= 140 - 121$

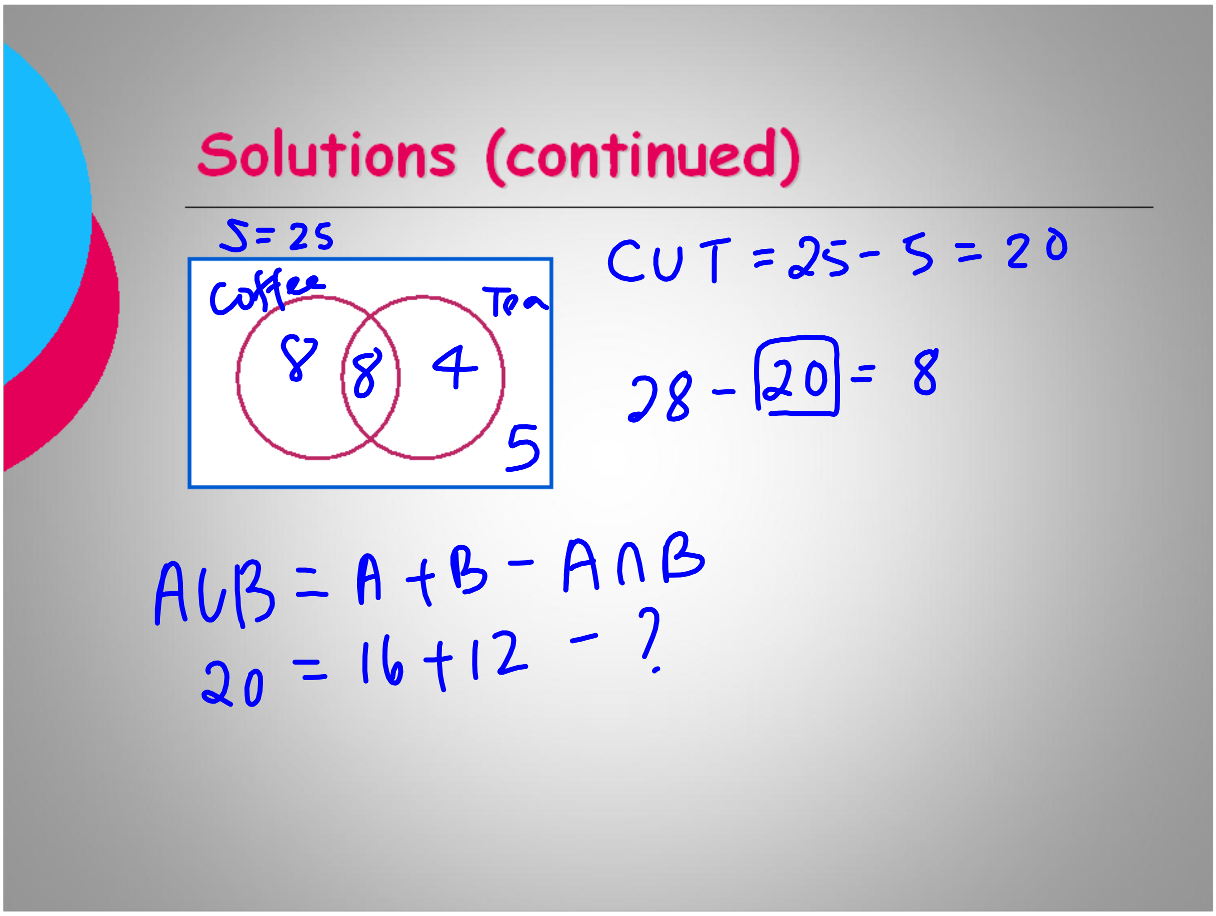
$= 19$

Examples (continued)

3. There were 25 employees at a meeting. Coffee and tea was served. Of the employees, 16 of them liked coffee, 12 of them liked tea, and 5 of them liked neither coffee nor tea. How many liked both?

Solutions (continued)




Solutions (continued)
S = 25
Coffee
Tea
8
8
4
5
CUT = 25 - 5 = 20
28 - 20 = 8
A∪B = A + B - A∩B
20 = 16 + 12 - ?

MDM4U **Organized Counting with Venn Diagrams** **Date:** ________

A **set** is a collection of distinct objects. We often notate sets with capital letters.

e.g. $A = \{x \mid x \geq 5\}$ $B = \{a, b, c, d\}$ $C = \emptyset = \{\ \}$ or empty set $D = \{0\}$

n(A) = infinite n(B) = 4 ∅ n(C) = 0 n(D) = 1

The objects *a,b,c,d* are called __elements__ or __members__ of the set B.

Note: Set C does not have any element and is called **empty set** or **null set.**

Does set D represent a null set?

No, Set D has one element, "0". odd even

To denote the number of elements in a set A, we write **n(A)**.

$E = \{x \mid 5 \leq x \leq 10, x \in Z\}$

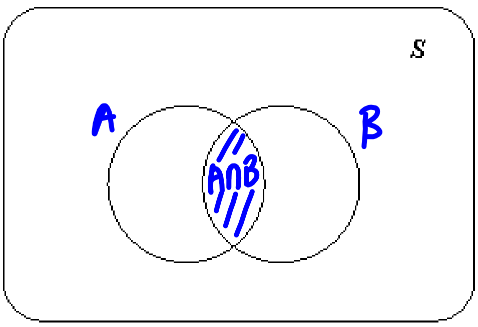
Relationships between sets and their subsets can be illustrated using **Venn diagrams** in which sets are represented by shaded or coloured geometrical shapes.

Intersection of Sets $A \cap B$

Given two sets, ***A*** and ***B***, if ***A*** and ***B*** overlap or have elements in common, the set of common elements is called the intersection of ***A*** and ***B***, and is written as $A \cap B$.

These common elements are members of set ***A*** and are also elements of set ***B***. Consequently, $A \cap B$ = {elements in both ***A*** **AND** ***B***}

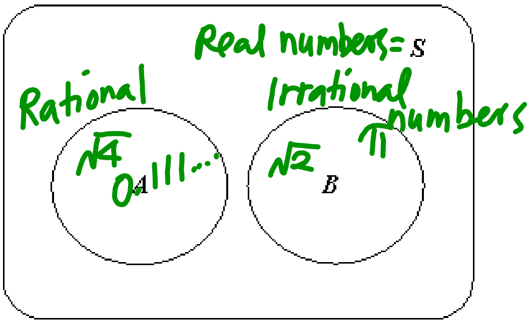
The set $A \cap B$ is represented by the region of overlap of the two sets in the Venn diagram below. Sets ***A*** and ***B*** exist as sets within the larger set ***S***, called the **universal set.** They are **subsets** of the set ***S***.



Disjoint Sets

If A and B have no elements in common (i.e., $n(A \cap B) = 0$), they are said to be disjoint and their intersection is the empty set, represented by the Greek letter $\varnothing$.
(i.e., $A \cap B = \varnothing$).

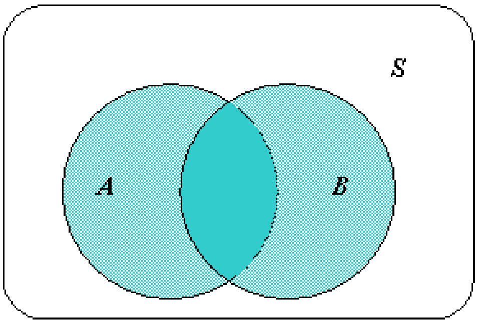
The diagram below shows disjoint sets A and B.



Union of Sets $A \cup B$

The set formed by combining the elements of A with those in B is called the union of A and B, and is written as $A \cup B$.
The elements in $A \cup B$ are elements of A or they are elements of B. Consequently, $A \cup B$ = {elements in A **OR** B}

The set $A \cup B$ is represented by the shaded area in the diagram below.



Example 1: Using a Venn Diagram to Solve a Counting Problem

Suppose a survey of 100 Grade 12 mathematics students in a local high school produced the following results.

Math Course Taken	Number of Students
Advanced Functions and Introductory Calculus	80
Geometry and Discrete Math	33
Data Management	68
Geometry and Discrete Math and Calculus	30
Geometry and Discrete Math and Data Management	6
Data Management and Calculus	50
All three courses	5

How many students are enrolled in Calculus and in no other mathematics course?

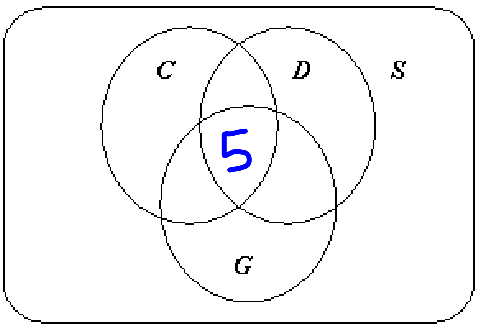
How many students are enrolled in Calculus? Or Data Management?

First construct a Venn diagram to represent the three groups of students. For convenience, we can label the sets as follow:
C for Calculus
G for Geometry and Discrete Math
D for Data Management.

The entire sample space, S, will consist of all the students in Grade 12.

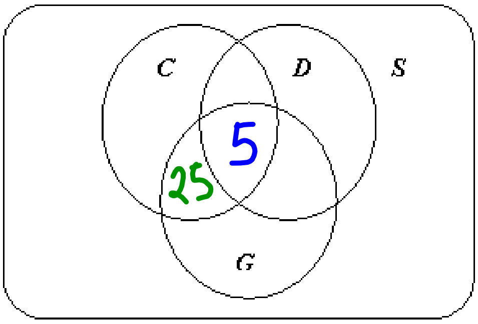
To avoid double counting, start entering information from the chart in the very middle of the diagram and work your way out.

$n(C \cap G \cap D) = 5$



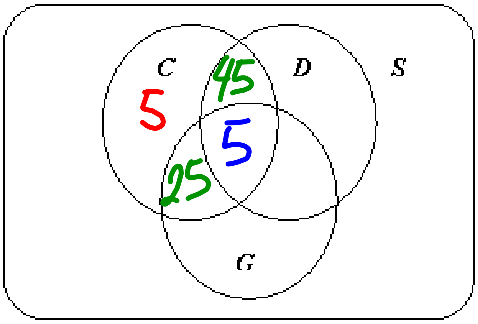
Now since $n(G \cap C) = 30$ and $n(C \cap G \cap D) = 5$

The number of students who take Calculus and Geometry but not Data Management must be 30 - 5 = 25



Since $n(D \cap C) = 50$ and $n(C \cap G \cap D) = 5$

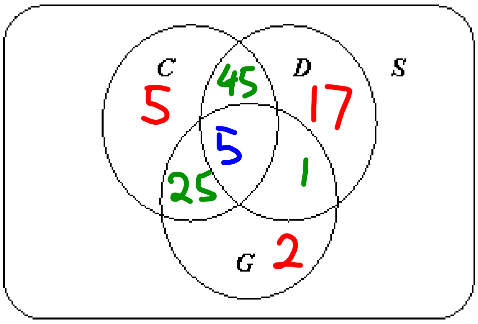
The number of students who take Calculus and Data but not Geometry must be 50 - 5 = 45.



There are 80 students enrolled in Calculus, of which 75 have been accounted for.

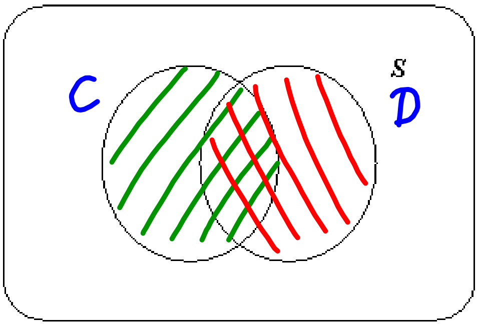
There must be 5 students who take only Calculus and no other mathematics course.

The complete Venn diagram:



If we consider only the students taking Calculus and Data Management, the Venn Diagram will have only two sets and one intersection. The total number of students in both courses is __50__.

$n(C \cup D) = n(C) + n(D) - n(C \cap D) =$ 80 + 68 - 50 = 98



Additive Principle for Unions of Two Sets

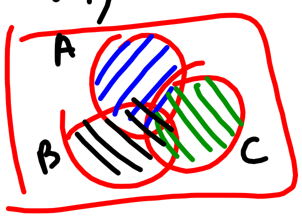
Given two sets, A and B, the number of elements in $A \cup B$ can be found by totalling the number of elements in both sets and then subtracting the number that have been counted twice. The double-counted elements will be found in the intersection of the two sets.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$A \cup B \cup C = A + B + C - A \cap B - A \cap C - B \cap C + A \cap B \cap C$
(union)

$A \cap B \cap C = A \cup B \cup C - (A + B + C) + A \cap B + A \cap C + B \cap C$
(Intersection)

$A \cap B \cap C = A \cap B + A \cap C + B \cap C - (A + B + C - A \cup B \cup C)$
(Intersection)



↑ This make more sense intuitively!