2.2 Indices

In the previous section, you used tables and graphs of frequencies to summarize data. Indices are another way to summarize data and recognize trends. An **index** relates the value of a variable (or group of variables) to a base level, which is often the value on a particular date. The base level is set so that the index produces numbers that are easy to understand and compare. Indices are used to report on a wide variety of variables, including prices and wages, ultraviolet levels in sunlight, and even the readability of textbooks.
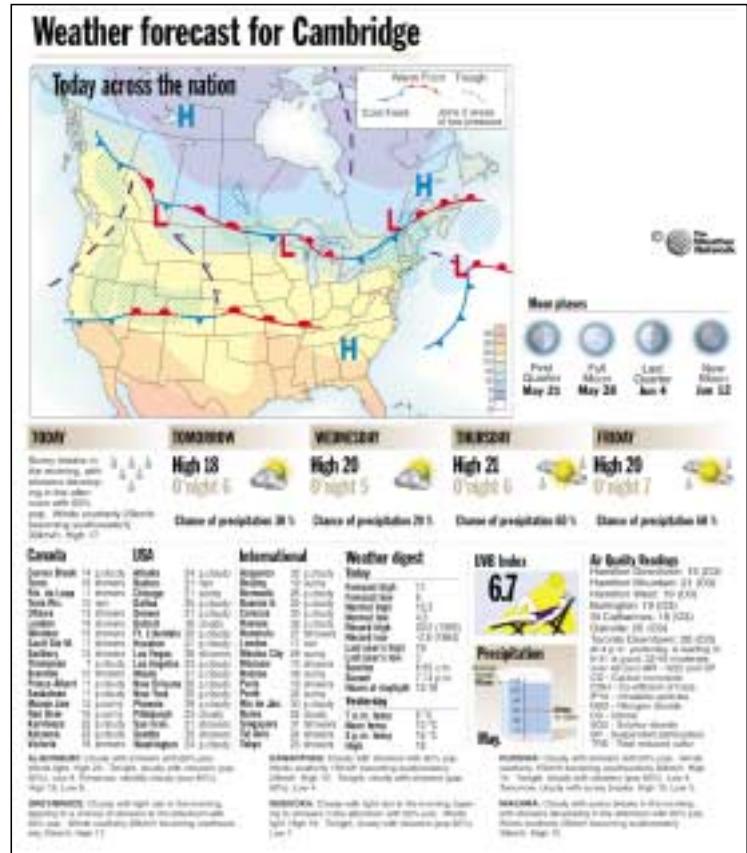
INVESTIGATE & INQUIRE: Consumer Price Index

The graph below shows Statistics Canada's consumer price index (CPI), which tracks the cost of over 600 items that would be purchased by a typical family in Canada. For this chart, the base is the cost of the same items in 1992.

1. What trend do you see in this graph? Estimate the annual rate of increase.

2. Estimate the annual rate of increase for the period from 1992 to 1996. Do you think the difference between this rate and the one from 1996 to 2001 is significant? Why or why not?

3. What was the index value in February of 1998? What does this value tell you about consumer prices at that time?

4. What would be the best way to estimate what the consumer price index will be in May of 2003? Explain your reasoning.

5. Explain how the choice of the vertical scale in the graph emphasizes changes in the index. Do you think this emphasis could be misleading? Why or why not?

The best-known Canadian business index is the S&P/TSX Composite Index, managed for the Toronto Stock Exchange by Standard & Poor's Corporation. Introduced in May, 2002, this index is a continuation of the TSE 300 Composite Index®, which goes back to 1977. The S&P/TSX Composite Index is a measure of the total market value of the shares of over 200 of the largest companies traded on the Toronto Stock Exchange. The index is the current value of these stocks divided by their total value in a base year and then multiplied by a scaling factor. When there are significant changes (such as takeovers or bankruptcies) in any of the companies in the index, the scaling factor is adjusted so that the values of the index remain directly comparable to earlier values. Note that the composite index weights each company by the total value of its shares (its market capitalization) rather than by the price of the individual shares. The S&P/TSX Composite Index usually indicates trends for major Canadian corporations reasonably well, but it does not always accurately reflect the overall Canadian stock market.

Time-series graphs are often used to show how indices change over time. Such graphs plot variable values versus time and join the adjacent data points with straight lines.

Example 1 Stock Market Index

The following table shows the TSE 300 Composite Index® from 1971 to 2001.
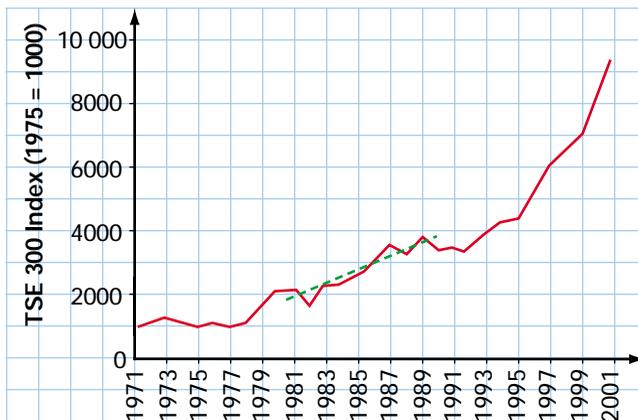
a) What does the notation "1975 = 1000" mean?

b) By what factor did the index grow over the period shown?

c) Estimate the rate of growth of the index during the 1980s.

Solution

a) The notation indicates that the index shows the stock prices *relative* to what
    they were in 1975. This 1975 base has been set at 1000. An index value of
    2000 would mean that overall the stocks of the 300 companies in the index
    are selling for twice what they did in 1975.

b) From the graph, you can see that the index increased from about 1000 in 1971
    to about 10 000 in 2001. Thus, the index increased by a factor of approximately
    10 over this period.

c) To estimate the rate of growth of the index during the 1980s, approximate
    the time-series graph with a straight line during that
    10-year interval. Then, calculate the slope of
    the line.

$$m = \frac{\text{rise}}{\text{run}}$$
$$\doteq \frac{3700 - 1700}{10}$$
$$= 200$$

The TSE 300 Composite Index® rose about 200 points a year during the 1980s.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

For more information on stock indices, visit the above
web site and follow the links. Write a brief
description of the rules for inclusion in
the various market indices.

Statistics Canada calculates a variety of carefully researched economic indices.
For example, there are price indices for new housing, raw materials, machinery
and equipment, industrial products, and farm products. Most of these indices
are available with breakdowns by province or region and by specific categories,
such as agriculture, forestry, or manufacturing. Statisticians, economists, and the
media make extensive use of these indices. (See section 1.3 for information on
how to access Statistics Canada data.)

The **consumer price index (CPI)** is the most widely reported of these
economic indices because it is an important measure of inflation. **Inflation** is
a general increase in prices, which corresponds to a decrease in the value of
money. To measure the average change in retail prices across Canada,
Statistics Canada monitors the retail prices of a set of over 600 goods and
services including food, shelter, clothing, transportation, household items,
health and personal care, recreation and education, and alcohol and tobacco
products. These items are representative of purchases by typical Canadians
and are weighted according to estimates of the total amount Canadians spend
on each item. For example, milk has a weighting of 0.69% while tea has a
weighting of only 0.06%.

Data in Action

Statistics Canada
usually publishes the
consumer price index
for each month in
the third week of the
following month.
Over 60 000 price
quotations are
collected for each
update.

The following graph shows the amount by which the consumer price index changed since the same month of the previous year.
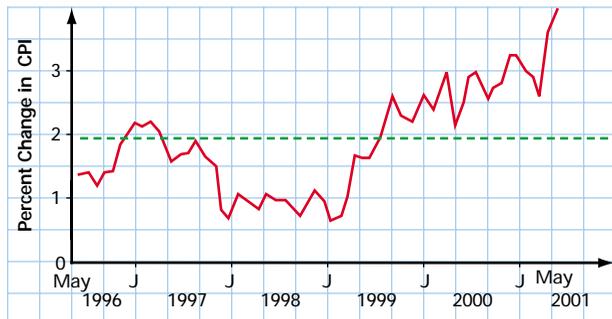
a) What does this graph tell you about changes in the CPI from 1996 to 2001?

b) Estimate the mean annual change in the CPI for this period.

Solution

a) Note that the graph above shows the annual changes in the CPI, unlike the graph on page 104, which illustrates the value of the CPI for any given month. From the above graph, you can see that the annual change in the CPI varied between 0.5% and 4% from 1996 to 2001. Overall, there is an upward trend in the annual change during this period.

b) You can estimate the mean annual change by drawing a horizontal line such that the total area between the line and the parts of the curve above it is approximately equal to the total area between the line and the parts of the curve below it. As shown above, this line meets the y-axis near 2%.

Thus, the mean annual increase in the CPI was roughly 2% from 1996 to 2001.

Project Prep

If your statistics project examines how a variable changes over time, a time-series graph may be an effective way to illustrate your findings.

The consumer price index and the cost of living index are not quite the same. The cost of living index measures the cost of maintaining a constant standard of living. If consumers like two similar products equally well, their standard of living does not change when they switch from one to the other. For example, if you like both apples and pears, you might start buying more apples and fewer pears if the price of pears went up while the price of apples was unchanged. Thus, your cost of living index increases less than the consumer price index does.

W E B C O N N E C T I O N

www.mcgrawhill.ca/links/MDM12

For more information about Statistics Canada indices, visit the above web site and follow the links to Statistics Canada.

Indices are also used in many other fields, including science, sociology, medicine, and engineering. There are even indices of the clarity of writing.

Example 3 Readability Index

The Gunning fog index is a measure of the readability of prose. This index estimates the years of schooling required to read the material easily.

Gunning fog index = 0.4(average words per sentence + percent "hard" words)

where "hard" words are all words over two syllables long except proper nouns, compounds of easy words, and verbs whose third syllable is *ed* or *es*.

a) Calculate the Gunning fog index for a book with an average sentence length of 8 words and a 20% proportion of hard words.

b) What are the advantages and limitations of this index?

Solution

a) Gunning fog index = 0.4(8 + 20)
$$= 11.2$$

The Gunning fog index shows that the book is written at a level appropriate for readers who have completed grade 11.

b) The Gunning fog index is easy to use and understand. It generates a grade-level rating, which is often more useful than a readability rating on an arbitrary scale, such as 1 to 10 or 1 to 100. However, the index assumes that bigger words and longer sentences always make prose harder to read. A talented writer could use longer words and sentences and still be more readable than writers who cannot clearly express their ideas. The Gunning fog index cannot, of course, evaluate literary merit.

Project Prep

You may want to use an index to summarize and compare sets of data in your statistics project.

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

Visit the above web site to find a link to a readability-index calculator. Determine the reading level of a novel of your choice.

Practise

A

1. Refer to the consumer price index graph on page 104.

   a) By how many index points did the CPI increase from January, 1992 to January, 2000?

   b) Express this increase as a percent.

   c) Estimate what an item that cost
      i) $7.50 in 1992 cost in April, 1998
      ii) $55 in August, 1997 cost in May, 2000

Apply, Solve, Communicate

2. a) Explain why there is a wide variety of items in the CPI basket.

   b) Is the percent increase for the price of each item in the CPI basket the same? Explain.

B

3. Refer to the graph of the TSE 300 Composite Index® on page 105.

   a) When did this index first reach five times its base value?

   b) Estimate the growth rate of the index from 1971 to 1977. What does this growth rate suggest about the Canadian economy during this period?

   c) During what two-year period did the index grow most rapidly? Explain your answer.

   d) Could a straight line be a useful mathematical model for the TSE 300 Composite Index®? Explain why or why not.

4. Communication

   a) Define inflation.

   b) In what way do the consumer price index and the new housing price index provide a measure of inflation?

c) How would you expect these two indices to be related?

d) Why do you think that they would be related in this way?

5. Application Consider the following time-series graph for the consumer price index.
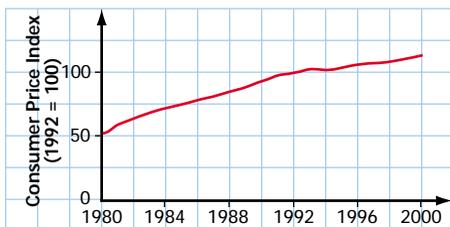
a) Identify at least three features of this graph that are different from the CPI graph on page 104.

b) Explain two advantages that the graph shown here has over the one on page 104.

c) Explain two disadvantages of the graph shown here compared to the one on page 104.

d) Estimate the year in which the CPI was at 50.

e) Explain the significance of the result in part d) in terms of prices in 1992.

6. Application The following graph illustrates the CPI both with and without energy price changes.
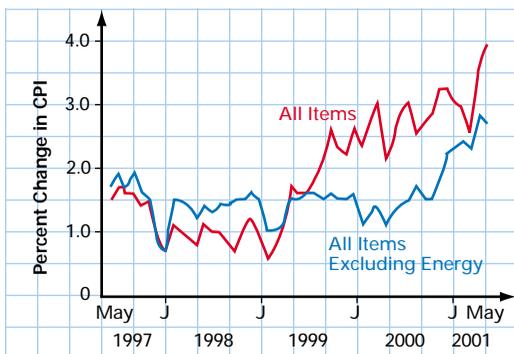
a) How is this graph different from the one on page 107?

b) Describe how the overall trend in energy costs compares to that of the CPI for the period shown.

c) What insight is gained by removing the energy component of the CPI?

d) Estimate the overall increase in the energy-adjusted CPI for the period shown.

e) Discuss how your result in part d) compares to the value found in part b) of Example 2.

7. François' agent wants to bargain for a better salary based on François' statistics for his first five seasons with the team.

a) Produce a time-series graph for François' goals, assists, and points over the past five years.

b) Calculate the mean number of goals, assists, and points per game played during each of François' five seasons.

c) Generate a new time-series graph based on the data from part b).

d) Which time-series graph will the agent likely use, and which will the team's manager likely use during the contract negotiations? Explain.

e) Explain the method or technology that you used to answer parts a) to d).

8. Aerial surveys of wolves in Algonquin Park produced the following estimates of their population density.

Year	Wolves/100 km²
1988–89	4.91
1989–90	2.47
1990–91	2.80
1991–92	3.62
1992–93	2.53
1993–94	2.23
1994–95	2.82
1995–96	2.75
1996–97	2.33
1997–98	3.04
1998–99	1.59

a) Using 1988–89 as a base, construct an index for these data.

b) Comment on any trends that you observe.

9. Use Statistics Canada web sites or other sources to find statistics for the following and describe any trends you notice.

a) the population of Canada

b) the national unemployment rate

c) the gross domestic product

10. Inquiry/Problem Solving

a) Use data from E-STAT or other sources to generate a time-series graph that shows the annual number of crimes in Canada for the period 1989–1999. If using E-STAT, look in the Nation section under Justice/Crimes and Offences.

b) Explain any patterns that you notice.

c) In what year did the number of crimes peak?

d) Suggest possible reasons why the number of crimes peaked in that year. What other statistics would you need to confirm whether these reasons are related to the peak in the number of crimes?

11. a) Use data from E-STAT or other sources to generate a time-series graph that shows the number of police officers in Canada for the period 1989–1999. If using E-STAT, look in the Nation section under Justice/Police services.

b) In what ways are the patterns in these data similar to the patterns in the data in question 10? In what ways are the patterns different?

c) In what year did the number of police officers peak?

d) Explain how this information could affect your answer to part d) of question 10.

12. Communication Use the Internet, a library, or other resources to research two indices not discussed in this section. Briefly describe what each index measures, recent trends in the index, and any explanation or rationale for these trends.

13. Inquiry/Problem Solving The pictograph below shows total greenhouse-gas emissions for each province and territory in 1996.

•	= 638
•	= 50 200
●	= 99 800
●	= 149 500
●	= 199 100

kilotonnes of CO_2 equivalent
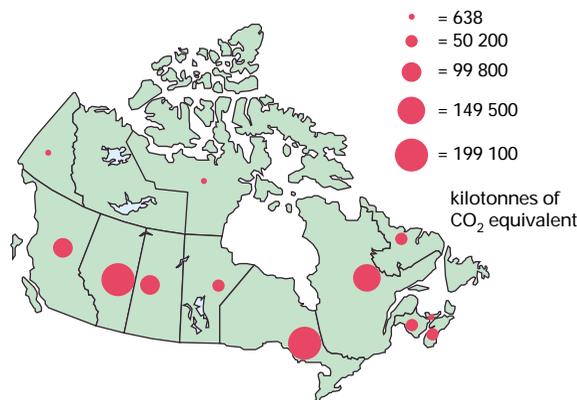
a) Which two provinces have the highest levels of greenhouse-gas emissions?

b) Are the diameters or areas of the circles proportional to the numbers they represent? Justify your answer.

c) What are the advantages and disadvantages of presenting these data as a pictograph?

d) Which provinces have the highest levels of greenhouse-gas emissions per geographic area?

e) Is your answer to part d) what you would have expected? How can you account for such relatively high levels in these areas?

f) Research information from E-STAT or other sources to determine the greenhouse-gas emissions per person for each province.

14. The graph below shows the national unemployment rate from January, 1997, to June, 2001.

a) Describe the overall trend for the period shown.

b) When did the unemployment rate reach its lowest level?

c) Estimate the overall unemployment rate for the period shown.

d) Explain what the term *seasonally adjusted* means.

e) Who is more likely to use this graph in an election campaign, the governing party or an opposing party? Explain.

f) How might an opposing party produce a graph showing rising unemployment without changing the data? Why would they produce such a graph?

C

15. A *Pareto chart* is a type of frequency diagram in which the frequencies for categorical data are shown by connected bars arranged in descending order of frequency. In a random survey, commuters listed their most common method of travelling to the downtown of a large city.

a) Construct a Pareto chart for these data.

b) Describe the similarities and differences between a Pareto chart and other frequency diagrams.

Method	Number of Respondents
Automobile: alone	26
Automobile: car pool	35
Bus/Streetcar	52
Train	40
Bicycle/Walking	13

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For more information about Pareto charts, visit the above web site and follow the links. Give two examples of situations where you would use a Pareto chart. Explain your reasoning.

16. Pick five careers of interest to you.

a) Use resources such as CANSIM II, E-STAT, newspapers, or the Internet to obtain information about entry-level income levels for these professions.

b) Choose an effective method to present your data.

c) Describe any significant information you discovered.

17. a) Research unemployment data for Ontario over the past 20 years.

b) Present the data in an appropriate form.

c) Conduct additional research to account for any trends or unusual features of the data.

d) Predict unemployment trends for both the short term and the long term. Explain your predictions.