Who will win the next federal election? Are Canadians concerned about global warming? Should a Canadian city bid to host the next Olympic Games? Governments, political parties, advocacy groups, and news agencies often want to know the public's opinions on such questions. Since it is not feasible to ask every citizen directly, researchers often survey a much smaller group and use the results to estimate the opinions of the entire population.


## INVESTIGATE \& INQUIRE: Extrapolating From a Sample

1. Work in groups or as a class to design a survey to determine the opinions of students in your school on a subject such as favourite movies, extra-curricular activities, or types of music.
2. Have everyone in your class answer the survey.
3. Decide how to categorize and record the results. Could you refine the survey questions to get results that are easier to work with? Explain the changes you would make.
4. How could you organize and present the data to make it easier to recognize any patterns? Can you draw any conclusions from the data?
5. a) Extrapolate your data to estimate the opinions of the entire school population. Explain your method.
b) Describe any reasons why you think the estimates in part a) may be inaccurate.
c) How could you improve your survey methods to get more valid results?

In statistics, the term population refers to all individuals who belong to a group being studied. In the investigation above, the population is all the students in your school, and your class is a sample of that population. The population for a statistical study depends on the kind of data being collected.

## Example 1 Identifying a Population

Identify the population for each of the following questions.
a) Whom do you plan to vote for in the next Ontario election?
b) What is your favourite type of baseball glove?
c) Do women prefer to wear ordinary glasses or contact lenses?

## Solution

a) The population consists of those people in Ontario who will be eligible to vote on election day.
b) The population would be just those people who play baseball. However, you might want to narrow the population further. For example, you might be interested only in answers from local or professional baseball players.
c) The population is all women who use corrective lenses.

Once you have identified the population, you need to decide how you will obtain your data. If the population is small, it may be possible to survey the entire group. For larger populations, you need to use an appropriate sampling technique. If selected carefully, a relatively small sample can give quite accurate results.

The group of individuals who actually have a chance of being selected is called the sampling frame. The sampling frame varies depending on the sampling technique used. Here are some of the most commonly used sampling techniques.

## Simple Random Sample

In a simple random sample, every member of the population has an equal chance of being selected and the selection of any particular individual does not affect the chances of any other individual being chosen. Choosing the sample randomly reduces the risk that selected members will not be representative of the whole population. You could select the sample by drawing names randomly or by assigning each member of the population a unique number and then using a random-number generator to determine which members to include.

## Systematic Sample

For a systematic sample, you go through the population sequentially and select members at regular intervals. The sample size and the population size determine the sampling interval.
interval $=\frac{\text { population size }}{\text { sample size }}$
For example, if you wanted the sample to be a tenth of the population, you would select every tenth member of the population, starting with one chosen randomly from among the first ten in sequence.

## Example 2 D esigning a Systematic Sample

A telephone company is planning a marketing survey of its 760000 customers. For budget reasons, the company wants a sample size of about 250 .
a) Suggest a method for selecting a systematic sample.
b) What expense is most likely to limit the sample size?

## Solution

a) First, determine the sampling interval.

$$
\begin{aligned}
\text { interval } & =\frac{\text { population size }}{\text { sample size }} \\
& =\frac{760000}{250} \\
& =3040
\end{aligned}
$$

The company could randomly select one of the first 3040 names on its list of customers and then choose every 3040th customer from that point on. For simplicity, the company might choose to select every 3000th customer instead.
b) The major cost is likely to be salaries for the staff to call and interview the customers.

## Stratified Sample

Sometimes a population includes groups of members who share common characteristics, such as gender, age, or education level. Such groups are called strata. A stratified sample has the same proportion of members from each stratum as the population does.

## Example 3 D esigning a Stratified Sample

Before booking bands for the school dances, the students' council at Statsville High School wants to survey the music preferences of the student body. The following table shows the enrolment at the school.

| Grade | Number of Students |
| :---: | :---: |
| 9 | 255 |
| 10 | 232 |
| 11 | 209 |
| 12 | 184 |
| Total | 880 |

a) Design a stratified sample for a survey of $25 \%$ of the student body.
b) Suggest other ways to stratify this sample.

## Solution

a) To obtain a stratified sample with the correct proportions, simply select $25 \%$ of the students in each grade level as shown on the right.

| Grade | Number of Students | Relative Frequency | Number Surveyed |
| :---: | :---: | :---: | :---: |
| 9 | 255 | 0.29 | 64 |
| 10 | 232 | 0.26 | 58 |
| 11 | 209 | 0.24 | 52 |
| 12 | 184 | 0.21 | 46 |
| Total | 880 | 1.00 | 220 |

b) The sample could be stratified according to gender or age instead of grade level.

## Other Sampling Techniques

C luster Sample: If certain groups are likely to be representative of the entire population, you can use a random selection of such groups as a cluster sample. For example, a fast-food chain could save time and money by surveying all its employees at randomly selected locations instead of surveying randomly selected employees throughout the chain.

M ulti-Stage Sample: A multi-stage sample uses several levels of random sampling. If, for example, your population consisted of all Ontario households, you could first randomly sample from all cities and townships in Ontario, then randomly sample from all subdivisions or blocks within the selected cities and townships, and finally randomly sample from all houses within the selected subdivisions or blocks.

Voluntary-R esponse Sample: In a voluntary-response sample, the researcher simply invites any member of the population to participate in the survey. The results from the responses of such surveys can be skewed because the people who choose to respond are often not representative of the population. Call-in shows and mail-in surveys rely on voluntary-response samples.

C onvenience Sample: Often, a sample is selected simply because it is easily accessible. While obviously not as random as some of the other techniques, such convenience samples can sometimes yield helpful information. The investigation at the beginning of this section used your class as a convenience sample.

## Key Concepts

- A carefully selected sample can provide accurate information about a population.
- Selecting an appropriate sampling technique is important to ensure that the sample reflects the characteristics of the population. Randomly selected samples have a good chance of being representative of the population.
- The choice of sampling technique will depend on a number of factors, such as the nature of the population, cost, convenience, and reliability.


## Communicate Your Understanding

1. What are the advantages and disadvantages of using a sample to estimate the characteristics of a population?
2. Discuss whether a systematic sample is a random sample.
3. a) Explain the difference between stratified sampling and cluster sampling.
b) Suggest a situation in which it would be appropriate to use each of these two sampling techniques.

## Practise

## A

1. Identify the population for each of the following questions.
a) Who should be the next president of the students' council?
b) Who should be next year's grade-10 representative on the student council?
c) What is the your favourite soft drink?
d) Which Beatles song was the best?
e) How effective is a new headache remedy?
2. Classify the sampling method used in each of the following scenarios.
a) A radio-show host invites listeners to call in with their views on banning smoking in restaurants.
b) The Heritage Ministry selects a sample of recent immigrants such that the proportions from each country of origin are the same as for all immigrants last year.
c) A reporter stops people on a downtown street to ask what they think of the city's lakefront.
d) A school guidance counsellor arranges interviews with every fifth student on the alphabetized attendance roster.
e) A statistician conducting a survey randomly selects 20 cities from across Canada, then 5 neighbourhoods from each of the cities, and then 3 households from each of the neighbourhoods.
f) The province randomly chooses 25 public schools to participate in a new fundraising initiative.
3. What type(s) of sample would be appropriate for
a) a survey of engineers, technicians, and managers employed by a company?
b) determining the most popular pizza topping?
c) measuring customer satisfaction for a department store?

## Apply, Solve, Communicate

## $B$

4. Natasha is organizing the annual family picnic and wants to arrange a menu that will appeal to children, teens, and adults. She estimates that she has enough time to survey about a dozen people. How should Natasha design a stratified sample if she expects 13 children, 8 teens, and 16 adults to attend the picnic?
5. Communication Find out, or estimate, how many students attend your school. Describe how you would design a systematic sample of these students. Assume that you can survey about 20 students.
6. The newly elected Chancellor of the Galactic Federation is interested in the opinions of all citizens regarding economic conditions in the galaxy. Unfortunately, she does not have the resources to visit every populated planet or to send delegates to them. Describe how the Chancellor might organize a multi-stage sample to carry out her survey.
7. Communication A community centre chooses 15 of its members at random and asks them to have each member of their families complete a short questionnaire.
a) What type of sample is the community centre using?
b) Are the 15 community-centre members a random sample of the community? Explain.
c) To what extent are the family members randomly chosen?
8. Application A students' council is conducting a poll of students as they enter the cafeteria.
a) What sampling method is the student council using?
b) Discuss whether this method is appropriate for surveying students' opinions on
i) the new mural in the cafeteria
ii) the location for the graduation prom
c) Would another sampling technique be better for either of the surveys in part b)?
9. Application The host of a call-in program invites listeners to comment on a recent trade by the Toronto Maple Leafs. One caller criticizes the host, stating that the sampling technique is not random. The host replies: "So what? It doesn't matter!"
a) What sampling technique is the call-in show using?
b) Is the caller's statement correct? Explain.
c) Is the host's response mathematically correct? Why or why not?

## C

10. Look in newspapers and periodicals or on the Internet for an article about a study involving a systematic, stratified, cluster, or multi-stage sample. Comment on the suitability of the sampling technique and the validity of the study. Present your answer in the form of a brief report. Include any suggestions you have for improving the study.
11. Inquiry/ Problem Solving Design a datagathering method that uses a combination of convenience and systematic sampling techniques.
12. Inquiry/ Problem Solving Pick a professional sport that has championship playoffs each year.
a) Design a multi-stage sample to gather your schoolmates' opinions on which team is likely to win the next championship.
b) Describe how you would carry out your study and illustrate your findings.
c) Research the media to find what the professional commentators are predicting. Do you think these opinions would be more valid than the results of your survey? Why or why not?
