

MDM4U

Measures of Central Tendency

Date: _____

What does average mean? In math terms, you have always been told that average is the sum of the data divided by the number of data items. In everyday terms, we are told average means ordinary, middle of the road, typical. The truth is that even in mathematics, the average or measure of central tendency can mean any of these things. It should mean the single value that best represents the data.

Average #1a: The Mean

This is what we typically think of as the average in math : $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

Ex. $x_1, x_2, x_3, x_4, x_5, \dots$
 12, 23, 24, 21, 2, 6 $\bar{x} = \frac{12 + 23 + 24 + 21 + 2 + 6}{6} = 14.67$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Average # 1b: The Weighted Mean

This is used to calculate the mean when you have multiple values or different weightings for each

measure of interest: $\bar{x} = \frac{\sum_{i=1}^n x_i w_i}{\sum_{i=1}^n w_i} \rightarrow n$

Ex. 4 64's, 3 24's and 2 90's $\bar{x} = \frac{64 \times 4 + 24 \times 3 + 90 \times 2}{4 + 3 + 2} = 56.44$

Score	Frequency
64	4
24	3
90	2
n = 9	

64 64 64 64
 24 24 24
 90 90

Average #1c: Grouped Data

This is used when you are given only a frequency table with ranges of data. Use the **midpoint** of each

interval for your weighted mean: $\bar{x} = \frac{\sum f \times m}{\sum f}$

Ex. A sample of car owners was asked how old they were when they got their first car. The results were then reported in a frequency distribution. Calculate the mean.

Age	16-20	21-25	26-30	31-35	36-40
Frequency	10	18	12	8	2

Median = 23
25th & 26th

Solution

$10 + 18 = 28$ $n = 50$

Age	Frequency, f	Midpoint (Age), m	f × m
16-20	10	18	180
21-25	18	23	414
26-30	12	28	336
31-35	8	33	264
36-40	2	38	76

Total n $18 \times 10 + 23 \times 18 + 28 \times 12 + 33 \times 8 + 38 \times 2$

$$\bar{x} = \frac{\sum f \times m}{\sum f} = \frac{180 + 414 + 336 + 264 + 76}{10 + 18 + 12 + 8 + 2} = 25.4$$

$n = \text{total}$

Average #2: The Median

We are already familiar with this measure. Simply sort the data values – the median is the value in the middle (odd number of items) or the mean of the two middle values (even number of items).

Ex. 3, 4, 5, 5, 5, 7, 7, 8, 9, 9 median = $\frac{5+7}{2} = 6$

least \longleftrightarrow greatest

Size	Frequency
3	1
4	1
5	3 → 5 th
7	2 → 6 th
8	1
9	2
10	

$$\frac{5+7}{2} = 6$$

Average #3: The Mode

This is the data value or interval that appears most frequently. It is easy to pick out of a histogram as the highest bar. It is also the only way to determine the “average” or typical response from qualitative data.

Ex. 3, 4, 5, 5, 5, 7, 7, 8, 9, 9 mode = 5

Can there be no mode for a set of data? Yes (when no individual data repeats)

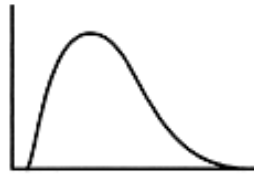
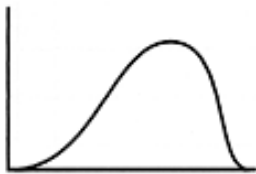
Can there be more than one mode? Yes

Which one do I use?

Mean – when data is roughly symmetric (mound or uniform) or sample size is big



Median – when data contains outliers or is heavily skewed or sample size is small



Mode – when data is qualitative, frequency is more important or data is bi-modal (u-shaped) or categorical

