Review of Key Concepts

5.1 Organized Counting With Venn Diagrams

Refer to the Key Concepts on page 270.

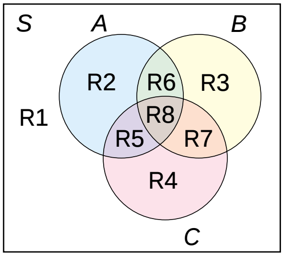
1. Which regions in the diagram below correspond to

a) the union of sets A and B?

b) the intersection of sets B and C?

c) $A \cap C$?

d) either B or S?



2. a) Write the equation for the number of elements contained in either of two sets.

b) Explain why the principle of inclusion and exclusion subtracts the last term in this equation.

c) Give a simple example to illustrate your explanation.

3. A survey of households in a major city found that

- 96% had colour televisions
- 65% had computers
- 51% had dishwashers
- 63% had colour televisions and computers
- 49% had colour televisions and dishwashers
- 31% had computers and dishwashers
- 30% had all three

a) List the categories of households not included in these survey results.

b) Use a Venn diagram to find the proportion of households in each of these categories.

5.2 Combinations

Refer to the Key Concepts on page 278.

4. Evaluate the following and indicate any calculations that could be done manually.

a) ${}_{41}C_8$ **b)** ${}_{33}C_{15}$

c) ${}_{25}C_{17}$ **d)** ${}_{50}C_{10}$

e) ${}_{10}C_8$ **f)** ${}_{15}C_{13}$

g) ${}_{5}C_4$ **h)** ${}_{25}C_{24}$

i) ${}_{15}C_{11}$ **j)** ${}_{25}C_{20}$

k) ${}_{16}C_8$ **l)** ${}_{30}C_{26}$

5. A track and field club has 12 members who are runners and 10 members who specialize in field events. The club has been invited to send a team of 3 runners and 2 field athletes to an out-of-town meet. How many different teams could the club send?

6. A bridge hand consists of 13 cards. How many bridge hands include 5 cards of one suit, 6 cards of a second, and 2 cards of a third?

7. Explain why combination locks should really be called permutation locks.

5.3 Problem Solving With Combinations

Refer to the Key Concepts on page 286.

8. At Subs Galore, you have a choice of lettuce, onions, tomatoes, green peppers, mushrooms, cheese, olives, cucumbers, and hot peppers on your submarine sandwich. How many ways can you “dress” your sandwich?

9. Ballots for municipal elections usually list candidates for several different positions. If a resident can vote for a mayor, two councillors, a school trustee, and a hydro commissioner, how many combinations of positions could the resident choose to mark on the ballot?

10. There are 12 questions on an examination, and each student must answer 8 questions including at least 4 of the first 5 questions. How many different combinations of questions could a student choose to answer?

11. Naomi invites eight friends to a party on short notice, so they may not all be able to come. How many combinations of guests could attend the party?

12. In how many ways could 15 different books be divided equally among 3 people?

13. The camera club has five members, and the mathematics club has eight. There is only one member common to both clubs. In how many ways could a committee of four people be formed with at least one member from each club?

5.4 The Binomial Theorem

Refer to the Key Concepts on page 293.

14. Without expanding $(x + y)^5$, determine
   a) the number of terms in the expansion
   b) the value of k in the term $10x^ky^2$

15. Use Pascal's triangle to expand
   a) $(x + y)^8$
   b) $(4x - y)^6$
   c) $(2x + 5y)^4$
   d) $(7x - 3)^5$

16. Use the binomial theorem to expand
   a) $(x + y)^6$
   b) $(6x - 5y)^4$
   c) $(5x + 2y)^5$
   d) $(3x - 2)^6$

17. Write the first three terms of the expansion of
   a) $(2x + 5y)^7$
   b) $(4x - y)^6$

18. Describe the steps in the binomial expansion of $(2x - 3y)^6$.

19. Find the last term in the binomial expansion of $\left(\frac{1}{x^2} + 2x\right)^5$.

20. Find the middle term in the binomial expansion of $\left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^8$.

21. In the expansion of $(a + x)^6$, the first three terms are $1 + 3 + 3.75$. Find the values of a and x.

22. Use the binomial theorem to expand and simplify $(y^2 - 2)^6(y^2 + 2)^6$.

23. Write $1024x^{10} - 3840x^8 + 5760x^6 - 4320x^4 + 1620x^2 - 243$ in the form $(a + b)^n$. Explain your steps.