# 6.2 Odds

**Odds** are another way to express a level of confidence about an outcome. Odds are commonly used in sports and other areas. Odds are often used when the probability of an event versus its complement is of interest, for example whether a sprinter will win or lose a race or whether a basketball team will make it to the finals.

## INVESTIGATE & INQUIRE: Tennis Tournament

For an upcoming tennis tournament, a television commentator estimates that the top-seeded (highest-ranked) player has "a 25% probability of winning, but her odds of winning are only 1 to 3."

1. a) If event $A$ is the top-seeded player winning the tournament, what is $A'$?
   b) Determine $P(A')$.

2. a) How are the odds of the top-seeded player winning related to $P(A)$ and $P(A')$?
   b) Should the television commentator be surprised that the odds were *only* 1 to 3? Why or why not?

3. a) What factors might the commentator consider when estimating the probability of the top-seeded player winning the tournament?
   b) How accurate do you think the commentator's estimate is likely to be? Would you consider such an estimate primarily a classical, an empirical, or a subjective probability? Explain.

**WEB CONNECTION**

www.mcgrawhill.ca/links/MDM12

For more information about tennis rankings and other tennis statistics, visit the above web site and follow the links. Locate some statistics about a tennis player of your choice. Use odds to describe these statistics.

The **odds in favour** of an event's occurring are given by the ratio of the probability that the event will occur to the probability that it will not occur.

$$\text{odds in favour of } A = \frac{P(A)}{P(A')}$$

Giving odds in favour of an event is a common way to express a probability.

### Example 1 Determining Odds

A messy drawer contains three red socks, five white socks, and four black socks. What are the odds in favour of randomly drawing a red sock?

#### Solution

Let the event $A$ be drawing a red sock. The probability of this event is

$$P(A) = \frac{3}{12}$$
$$= \frac{1}{4}$$

The probability of not drawing a red sock is

$$P(A') = 1 - P(A)$$
$$= \frac{3}{4}$$

Using the definition of odds,

$$\text{odds in favour of } A = \frac{P(A)}{P(A')}$$
$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$
$$= \frac{1}{3}$$

Therefore, the odds in favour of drawing a red sock are $\frac{1}{3}$, or less than 1. You are more likely *not* to draw a red sock. These odds are commonly written as 1:3, which is read as "one to three" or "one in three."

**Project Prep**

A useful feature you could include in your probability project is a calculation of the odds of winning your game.

Notice in Example 1 that the ratio of red socks to other socks is 3:9, which is the same as the odds in favour of drawing a red sock. In fact, the odds in favour of an event $A$ can also be found using

$$\text{odds in favour of } A = \frac{n(A)}{n(A')}$$

A common variation on the theme of odds is to express the odds *against* an event happening.

$$\text{odds against } A = \frac{P(A')}{P(A)}$$

### Example 2 Odds Against an Event

If the chance of a snowstorm in Windsor, Ontario, in January is estimated at 0.4, what are the odds against Windsor's having a snowstorm next January? Is a January snowstorm more likely than not?

#### Solution

Let event $A$ = {snowstorm in January}.
Since $P(A) + P(A') = 1$,

$$\begin{aligned}\text{odds against } A &= \frac{P(A')}{P(A)} \\ &= \frac{1 - P(A)}{P(A)} \\ &= \frac{1 - 0.4}{0.4} \\ &= \frac{0.6}{0.4} \\ &= \frac{3}{2}\end{aligned}$$

The odds against a snowstorm are 3:2, which is greater than 1:1. So a snowstorm is *less* likely to occur than not.

Sometimes, you might need to convert an expression of odds into a probability. You can do this conversion by expressing $P(A')$ in terms of $P(A)$.

### Example 3 Probability From Odds

A university professor, in an effort to promote good attendance habits, states that the odds of passing her course are 8 to 1 when a student misses fewer than five classes. What is the probability that a student with good attendance will pass?

#### Solution

Let the event $A$ be that a student with good attendance passes. Since

$$\text{odds in favour of } A = \frac{P(A)}{P(A')},$$

$$\frac{8}{1} = \frac{P(A)}{P(A')}$$

$$= \frac{P(A)}{1 - P(A)}$$

$$8 - 8P(A) = P(A)$$

$$8 = 9P(A)$$

$$P(A) = \frac{8}{9}$$

The probability that a student with good attendance will pass is $\frac{8}{9}$, or approximately 89%.

In general, it can be shown that if the odds in favour of $A = \frac{h}{k}$, then $P(A) = \frac{h}{h+k}$.

### Example 4 Using the Odds-Probability Formula

The odds of Rico's hitting a home run are 2:7. What is the probability of Rico's hitting a home run?

#### Solution

Let $A$ be the event that Rico hits a home run. Then, $h = 2$ and $k = 7$, and

$$P(A) = \frac{h}{h+k}$$

$$= \frac{2}{2+7}$$

$$= \frac{2}{9}$$

Rico has approximately a 22% chance of hitting a home run.

### Key Concepts

- The odds in favour of $A$ are given by the ratio $\frac{P(A)}{P(A')}$.
- The odds against $A$ are given by the ratio $\frac{P(A')}{P(A)}$.
- If the odds in favour of $A$ are $\frac{h}{k}$, then $P(A) = \frac{h}{h+k}$.

## Communicate Your Understanding

1. Explain why the terms *odds* and *probability* have different meanings. Give an example to illustrate your answer.
2. Would you prefer the odds in favour of passing your next data management test to be 1:3 or 3:1? Explain your choice.
3. Explain why odds can be greater than 1, but probabilities must be between 0 and 1.

## Practise

A

1. Suppose the odds in favour of good weather tomorrow are 3:2.
   a) What are the odds against good weather tomorrow?
   b) What is the probability of good weather tomorrow?
2. The odds against the Toronto Argonauts winning the Grey Cup are estimated at 19:1. What is the probability that the Argos will win the cup?
3. Determine the odds in favour of rolling each of the following sums with a standard pair of dice.
   a) 12
   b) 5 or less
   c) a prime number
   d) 1
4. Calculate the odds in favour of each event.
   a) New Year's Day falling on a Friday
   b) tossing three tails with three coins
   c) not tossing exactly two heads with three coins
   d) randomly drawing a black 6 from a complete deck of 52 cards
   e) a random number from 1 to 9 inclusive being even

## Apply, Solve, Communicate

B

5. Greta's T-shirt drawer contains three tank tops, six V-neck T-shirts, and two sleeveless shirts. If she randomly draws a shirt from the drawer, what are the odds that she will
   a) draw a V-neck T-shirt?
   b) not draw a tank top?
6. **Application** If the odds in favour of Boris beating Elena in a chess game are 5 to 4, what is the probability that Elena will win an upset victory in a best-of-five chess tournament?
7. Chapter Problem
   a) Based on the randomly tagged sample, what are the odds in favour of a captured deer being a cross-hatched buck?
   b) What are the odds against capturing a doe?

WEB CONNECTION

www.mcgrawhill.ca/links/MDM12

Visit the above web site and follow the links for more information about Canadian wildlife.

8. The odds against $A$, by definition, are equivalent to the odds in favour of $A'$. Use this definition to show that the odds against $A$ are equal to the reciprocal of the odds in favour of $A$.

9. **Application** Suppose the odds of the Toronto Maple Leafs winning the Stanley Cup are 1:5, while the odds of the Montréal Canadiens winning the Stanley Cup are 2:13. What are the odds in favour of either Toronto or Montréal winning the Stanley Cup?

10. What are the odds against drawing
    a) a face card from a standard deck?
    b) two face cards?

**ACHIEVEMENT CHECK**

| Knowledge / Understanding | Thinking / Inquiry / Problem Solving | Communication | Application |
|---|---|---|---|

11. Mike has a loaded (or unfair) six-sided die. He rolls the die 200 times and determines the following probabilities for each score:

$P(1) = 0.11$
$P(2) = 0.02$
$P(3) = 0.18$
$P(4) = 0.21$
$P(5) = 0.40$

    a) What is $P(6)$?
    b) Mike claims that the odds in favour of tossing a prime number with this die are the same as with a fair die. Do you agree with his claim?
    c) Using Mike's die, devise a game with odds in Mike's favour that an unsuspecting person would be tempted to play. Use probabilities to show that the game is in Mike's favour. Explain why a person who does not realize that the die is loaded might be tempted by this game.

12. George estimates that there is a 30% chance of rain the next day if he waters the lawn, a 40% chance if he washes the car, and a 50% chance if he plans a trip to the beach. Assuming George's estimates are accurate, what are the odds
    a) in favour of rain tomorrow if he waters the lawn?
    b) in favour of rain tomorrow if he washes the car?
    c) against rain tomorrow if he plans a trip to the beach?

C

13. **Communication** A volleyball coach claims that at the next game, the odds of her team winning are 3:1, the odds against losing are 5:1, and the odds against a tie are 7:1. Are these odds possible? Explain your reasoning.

14. **Inquiry / Problem Solving** Aki is a participant on a trivia-based game show. He has an equal likelihood on any given trial of being asked a question from one of six categories: Hollywood, Strange Places, Number Fun, Who?, Having a Ball, and Write On! Aki feels that he has a 50/50 chance of getting Having a Ball or Strange Places questions correct, but thinks he has a 90% probability of getting any of the other questions right. If Aki has to get two of three questions correct, what are his odds of winning?

15. **Inquiry / Problem Solving** Use logic and mathematical reasoning to show that if the odds in favour of $A$ are given by $\frac{h}{k}$, then $P(A) = \frac{h}{h+k}$. Support your reasoning with an example.