6.5 Mutually Exclusive Events

The phone rings. Jacques is really hoping that it is one of his friends calling about either softball or band practice. Could the call be about both?

In such situations, more than one event could occur during a single trial. You need to compare the events in terms of the outcomes that make them up. What is the chance that at least one of the events happens? Is the situation "either/or," or can both events occur?

INVESTIGATE & INQUIRE: Baseball Pitches

Marie, at bat for the Coyotes, is facing Anton, who is pitching for the Power Trippers. Anton uses three pitches: a fastball, a curveball, and a slider. Marie feels she has a good chance of making a base hit, or better, if Anton throws either a fastball or a slider. The count is two strikes and three balls. In such full-count situations, Anton goes to his curveball one third of the time, his slider half as often, and his fastball the rest of the time.

1. Determine the probability of Anton throwing his
   a) curveball b) slider c) fastball
2. a) What is the probability that Marie will get the pitch she does not want?
   b) Explain how you can use this information to determine the probability that Marie will get a pitch she likes.
3. a) Show another method of determining this probability.
   b) Explain your method.
4. What do your answers to questions 2 and 3 suggest about the probabilities of events that cannot happen simultaneously?

The possible events in this investigation are said to be **mutually exclusive** (or **disjoint**) since they cannot occur at the same time. The pitch could not be both a fastball and a slider, for example. In this particular problem, you were interested in the probability of *either* of two favourable events. You can use the notation $P(A \text{ or } B)$ to stand for the probability of either A or B occurring.

Example 1 Probability of Mutually Exclusive Events

Teri attends a fundraiser at which 15 T-shirts are being given away as door prizes. Door prize winners are randomly given a shirt from a stock of 2 black shirts, 4 blue shirts, and 9 white shirts. Teri really likes the black and blue shirts, but is not too keen on the white ones. Assuming that Teri wins the first door prize, what is the probability that she will get a shirt that she likes?

Solution

Let A be the event that Teri wins a black shirt, and B be the event that she wins a blue shirt.

$P(A) = \frac{2}{15}$ and $P(B) = \frac{4}{15}$

Teri would be happy if either A or B occurred.
There are $2 + 4 = 6$ non-white shirts, so

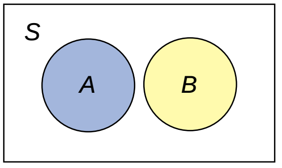
$$P(A \text{ or } B) = \frac{6}{15} = \frac{2}{5}$$

The probability of Teri winning a shirt that she likes is $\frac{2}{5}$ or 40%. Notice that this probability is simply the sum of the probabilities of the two mutually exclusive events.

When events A and B are mutually exclusive, the probability that A or B will occur is given by the **addition rule for mutually exclusive events**:

$$P(A \text{ or } B) = P(A) + P(B)$$

A Venn diagram shows mutually exclusive events as non-overlapping, or disjoint. Thus, you can apply the additive counting principle (see Chapter 4) to prove this rule.



Proof:

If A and B are mutually exclusive events, then

$$\begin{aligned} P(A \text{ or } B) &= \frac{n(A \text{ or } B)}{n(S)} \\ &= \frac{n(A) + n(B)}{n(S)} \\ &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} \\ &= P(A) + P(B) \end{aligned}$$

A and B are disjoint sets, and thus share no elements.

In some situations, events are **non-mutually exclusive**, which means that they can occur simultaneously. For example, consider a board game in which you need to roll either an 8 or doubles, using two dice.

Notice that in one outcome, rolling two fours, both events have occurred simultaneously. Hence, these events are not mutually exclusive. Counting the outcomes in the diagram shows that the probability of rolling either an 8 or doubles is $\frac{10}{36}$ or $\frac{5}{18}$. You need to take care not to count the (4, 4) outcome twice. You are applying the principle of inclusion and exclusion, which was explained in greater detail in Chapter 5.

Second die

First die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example 2 Probability of Non-Mutually Exclusive Events

A card is randomly selected from a standard deck of cards. What is the probability that either a heart or a face card (jack, queen, or king) is selected?

Solution

Let event A be that a heart is selected, and event B be that a face card is selected.

$P(A) = \frac{13}{52}$ and $P(B) = \frac{12}{52}$

If you add these probabilities, you get

$$P(A) + P(B) = \frac{13}{52} + \frac{12}{52}$$
$$= \frac{25}{52}$$

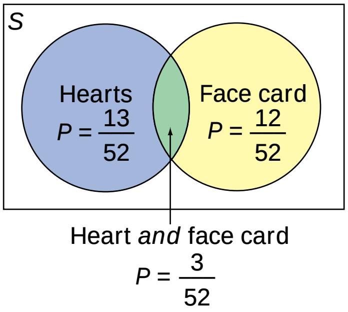
However, since the jack, queen, and king of hearts are in both A and B, the sum $P(A) + P(B)$ actually includes these outcomes *twice*.

A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣
A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦
A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥
A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠

Based on the diagram, the actual theoretical probability of drawing either a heart or a face card is $\frac{22}{52}$, or $\frac{11}{26}$. You can find the correct value by subtracting the probability of selecting the three elements that were counted twice.

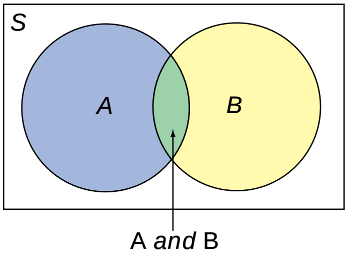
$$P(A \text{ or } B) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$
$$= \frac{22}{52}$$
$$= \frac{11}{26}$$

The probability that either a heart or a face card is selected is $\frac{11}{26}$.



When events A and B are non-mutually exclusive, the probability that A or B will occur is given by the **addition rule for non-mutually exclusive events**:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Example 3 Applying the Addition Rule for Non-Mutually Exclusive Events

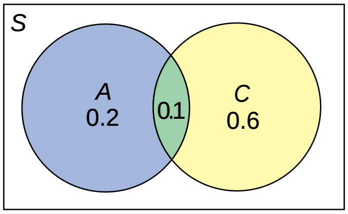
An electronics manufacturer is testing a new product to see whether it requires a surge protector. The tests show that a voltage spike has a 0.2% probability of damaging the product's power supply, a 0.6% probability of damaging downstream components, and a 0.1% probability of damaging both the power supply and other components. Determine the probability that a voltage spike will damage the product.

Project Prep

When analysing the possible outcomes for your game in the probability project, you may need to consider mutually exclusive or non-mutually exclusive events. If so, you will need to apply the appropriate addition rule to determine theoretical probabilities.

Solution

Let A be damage to the power supply and C be damage to other components.



The overlapping region represents the probability that a voltage surge damages both the power supply and another component. The probability that either A or C occurs is given by

$$P(A \text{ or } C) = P(A) + P(C) - P(A \text{ and } C)$$
$$= 0.2\% + 0.6\% - 0.1\%$$
$$= 0.7\%$$

There is a 0.7% probability that a voltage spike will damage the product.

Key Concepts

- If A and B are mutually exclusive events, then the probability of either A or B occurring is given by $P(A \text{ or } B) = P(A) + P(B)$.
- If A and B are non-mutually exclusive events, then the probability of either A or B occurring is given by $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Communicate Your Understanding

1. Are an event and its complement mutually exclusive? Explain.
2. Explain how to determine the probability of randomly throwing either a composite number or an odd number using a pair of dice.
3. a) Explain the difference between independent events and mutually exclusive events.
   b) Support your explanation with an example of each.
   c) Why do you add probabilities in one case and multiply them in the other?

Practise

A

1. Classify each pair of events as mutually exclusive or non-mutually exclusive.

	Event A	Event B
a)	Randomly drawing a grey sock from a drawer	Randomly drawing a wool sock from a drawer
b)	Randomly selecting a student with brown eyes	Randomly selecting a student on the honour roll
c)	Having an even number of students in your class	Having an odd number of students in your class
d)	Rolling a six with a die	Rolling a prime number with a die
e)	Your birthday falling on a Saturday next year	Your birthday falling on a weekend next year
f)	Getting an A on the next test	Passing the next test
g)	Calm weather at noon tomorrow	Stormy weather at noon tomorrow
h)	Sunny weather next week	Rainy weather next week

2. Nine members of a baseball team are randomly assigned field positions. There are three outfielders, four infielders, a pitcher, and a catcher. Troy is happy to play any position except catcher or outfielder. Determine the probability that Troy will be assigned to play
   a) catcher
   b) outfielder
   c) a position he does not like

3. A car dealership analysed its customer database and discovered that in the last model year, 28% of its customers chose a 2-door model, 46% chose a 4-door model, 19% chose a minivan, and 7% chose a 4-by-4 vehicle. If a customer was selected randomly from this database, what is the probability that the customer
   a) bought a 4-by-4 vehicle?
   b) did not buy a minivan?
   c) bought a 2-door or a 4-door model?
   d) bought a minivan or a 4-by-4 vehicle?

Apply, Solve, Communicate

B

4. As a promotion, a resort has a draw for free family day-passes. The resort considers July, August, March, and December to be “vacation months.”

a) If the free passes are randomly dated, what is the probability that a day-pass will be dated within

i) a vacation month?

ii) June, July, or August

b) Draw a Venn diagram of the events in part a).

5. A certain provincial park has 220 campsites. A total of 80 sites have electricity. Of the 52 sites on the lakeshore, 22 of them have electricity. If a site is selected at random, what is the probability that

a) it will be on the lakeshore?

b) it will have electricity?

c) it will either have electricity or be on the lakeshore?

d) it will be on the lakeshore and not have electricity?

6. A market-research firm monitored 1000 television viewers, consisting of 800 adults and 200 children, to evaluate a new comedy series that aired for the first time last week. Research indicated that 250 adults and 148 children viewed some or all of the program. If one of the 1000 viewers was selected, what is the probability that

a) the viewer was an adult who did not watch the new program?

b) the viewer was a child who watched the new program?

c) the viewer was an adult or someone who watched the new program?

7. **Application** In an animal-behaviour study, hamsters were tested with a number of intelligence tasks, as shown in the table below.

Number of Tests	Number of Hamsters
0	10
1	6
2	4
3	3
4 or more	5

If a hamster is randomly chosen from this study group, what is the likelihood that the hamster has participated in

a) exactly three tests?

b) fewer than two tests?

c) either one or two tests?

d) no tests or more than three tests?

8. **Communication**

a) Prove that, if A and B are non-mutually exclusive events, the probability of either A or B occurring is given by $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

b) What can you conclude if $P(A \text{ and } B) = 0$? Give reasons for your conclusion.

9. **Inquiry/Problem Solving** Design a game in which the probability of drawing a winning card from a standard deck is between 55% and 60%.

10. Determine the probability that a captured deer has either cross-hatched antlers or bald patches. Are these events mutually exclusive? Why or why not?

11. The eight members of the debating club pose for a yearbook photograph. If they line up randomly, what is the probability that

a) either Hania will be first in the row or Aaron will be last?

b) Hania will be first and Aaron will not be last?

ACHIEVEMENT CHECK

Knowledge / Understanding	Thinking / Inquiry / Problem Solving	Communication	Application

12. Consider a Stanley Cup playoff series in which the Toronto Maple Leafs hockey team faces the Ottawa Senators. Toronto hosts the first, second, and if needed, fifth and seventh games in this best-of-seven contest. The Leafs have a 65% chance of beating the Senators at home in the first game. After that, they have a 60% chance of a win at home if they won the previous game, but a 70% chance if they are bouncing back from a loss. Similarly, the Leafs' chances of victory in Ottawa are 40% after a win and 45% after a loss.

a) Construct a tree diagram to illustrate all the possible outcomes of the first three games.

b) Consider the following events:

A = {Leafs lose the first game but go on to win the series in the fifth game}

B = {Leafs win the series in the fifth game}

C = {Leafs lose the series in the fifth game}

Identify all the outcomes that make up each event, using strings of letters, such as *LLSLL*. Are any pairs from these three events mutually exclusive?

c) What is the probability of event A in part b)?

d) What is the chance of the Leafs winning in exactly five games?

e) Explain how you found your answers to parts c) and d).

C

13. A grade 12 student is selected at random to sit on a university liaison committee. Of the 120 students enrolled in the grade 12 university-preparation mathematics courses,

- 28 are enrolled in data management only
- 40 are enrolled in calculus only
- 15 are enrolled in geometry only
- 16 are enrolled in both data management and calculus
- 12 are enrolled in both calculus and geometry
- 6 are enrolled in both geometry and data management
- 3 are enrolled in all three of data management, calculus, and geometry

a) Draw a Venn diagram to illustrate this situation.

b) Determine the probability that the student selected will be enrolled in either data management or calculus.

c) Determine the probability that the student selected will be enrolled in only one of the three courses.

14. Application For a particular species of cat, the odds against a kitten being born with either blue eyes or white spots are 3:1. If the probability of a kitten exhibiting only one of these traits is equal and the probability of exhibiting both traits is 10%, what are the odds in favour of a kitten having blue eyes?

15. Communication

a) A standard deck of cards is shuffled and three cards are selected. What is the probability that the third card is either a red face card or a king if the king of diamonds and the king of spades are selected as the first two cards?

b) Does this probability change if the first two cards selected are the queen of diamonds and the king of spades? Explain.

16. **Inquiry/Problem Solving** The table below lists the degrees granted by Canadian universities from 1994 to 1998 in various fields of study.

a) If a Canadian university graduate from 1998 is chosen at random, what is the probability that the student is

i) a male?

ii) a graduate in mathematics and physical sciences?

iii) a male graduating in mathematics and physical sciences?

iv) not a male graduating in mathematics and physical sciences?

v) a male *or* a graduate in mathematics and physical sciences?

b) If a male graduate from 1996 is selected at random, what is the probability that he is graduating in mathematics and physical sciences?

c) If a mathematics and physical sciences graduate is selected at random from the period 1994 to 1996, what is the probability that the graduate is a male?

d) Do you think that being a male and graduating in mathematics and physical sciences are independent events? Give reasons for your hypothesis.

	1994	1995	1996	1997	1998
Canada	178 074	178 066	178 116	173 937	172 076
Male	76 470	76 022	75 106	73 041	71 949
Female	101 604	102 044	103 010	100 896	100 127
Social sciences	69 583	68 685	67 862	66 665	67 019
Male	30 700	29 741	29 029	28 421	27 993
Female	38 883	38 944	38 833	38 244	39 026
Education	30 369	30 643	29 792	27 807	25 956
Male	9093	9400	8693	8036	7565
Female	21 276	21 243	21 099	19 771	18 391
Humanities	23 071	22 511	22 357	21 373	20 816
Male	8427	8428	8277	8034	7589
Female	14 644	14 083	14 080	13 339	13 227
Health professions and occupations	12 183	12 473	12 895	13 073	12 658
Male	3475	3461	3517	3460	3514
Female	8708	9012	9378	9613	9144
Engineering and applied sciences	12 597	12 863	13 068	12 768	12 830
Male	10 285	10 284	10 446	10 125	10 121
Female	2312	2579	2622	2643	2709
Agriculture and biological sciences	10 087	10 501	11 400	11 775	12 209
Male	4309	4399	4756	4780	4779
Female	5778	6102	6644	6995	7430
Mathematics and physical sciences	9551	9879	9786	9738	9992
Male	6697	6941	6726	6749	6876
Female	2854	2938	3060	2989	3116
Fine and applied arts	5308	5240	5201	5206	5256
Male	1773	1740	1780	1706	1735
Female	3535	3500	3421	3500	3521
Arts and sciences	5325	5271	5755	5532	5340
Male	1711	1628	1882	1730	1777
Female	3614	3643	3873	3802	3563