6.6 Applying Matrices to Probability Problems

In some situations, the probability of an outcome depends on the outcome of the previous trial. Often this pattern appears in stock market trends, weather patterns, athletic performance, and consumer habits. Dependent probabilities can be calculated using Markov chains, a powerful probability model pioneered about a century ago by the Russian mathematician Andrei Markov.

INVESTIGATE & INQUIRE: Running Late

Although Marla tries hard to be punctual, the demands of her home life and the challenges of commuting sometimes cause her to be late for work. When she is late, she tries especially hard to be punctual the next day. Suppose that the following pattern emerges: If Marla is punctual on any given day, then there is a 70% chance that she will be punctual the next day and a 30% chance that she will be late. On days she is late, however, there is a 90% chance that she will be punctual the next day and just a 10% chance that she will be late. Suppose Marla is punctual on the first day of the work week.

1. Create a tree diagram of the possible outcomes for the second and third days. Show the probability for each branch.

2. a) Describe two branches in which Marla is punctual on day 3.
   b) Use the product rule for dependent events on page 332 to calculate the compound probability of Marla being punctual on day 2 and on day 3.
   c) Find the probability of Marla being late on day 2 and punctual on day 3.
   d) Use the results from parts b) and c) to determine the probability that Marla will be punctual on day 3.

3. Repeat question 2 for the outcome of Marla being late on day 3.

4. a) Create a 1×2 matrix A in which the first element is the probability that Marla is punctual and the second element is the probability that she is late on day 1. Recall that Marla is punctual on day 1.
   b) Create a 2×2 matrix B in which the elements in each row represent *conditional* probabilities that Marla will be punctual and late. Let the first row be the probabilities after a day in which Marla was punctual, and the second row be the probabilities after a day in which she was late.

c) Evaluate $A \times B$ and $A \times B^2$.

d) Compare the results of part c) with your answers to questions 2 and 3. Explain what you notice.

e) What does the first row of the matrix B^2 represent?

The matrix model you have just developed is an example of a **Markov chain**, a probability model in which the outcome of any trial depends directly on the outcome of the previous trial. Using matrix operations can simplify probability calculations, especially in determining long-term trends.

The 1×2 matrix A in the investigation is an **initial probability vector, $S^{(0)}$**, and represents the probabilities of the initial state of a Markov chain. The 2×2 matrix B is a **transition matrix, P**, and represents the probabilities of moving from any initial state to a new state in any trial.

These matrices have been arranged such that the product $S^{(0)} \times P$ generates the row matrix that gives the probabilities of each state after one trial. This matrix is called the **first-step probability vector, $S^{(1)}$**. In general, the ***n*th-step probability vector, $S^{(n)}$**, can be obtained by repeatedly multiplying the probability vector by P. Sometimes these vectors are also called **first-state** and ***n*th-state vectors**, respectively.

Notice that each entry in a probability vector or a transition matrix is a probability and must therefore be between 0 and 1. The possible states in a Markov chain are always mutually exclusive events, one of which must occur at each stage. Therefore, the entries in a probability vector must sum to 1, as must the entries in each row of the transition matrix.

Example 1 Probability Vectors

Two video stores, Video Vic's and MovieMaster, have just opened in a new residential area. Initially, they each have half of the market for rented movies. A customer who rents from Video Vic's has a 60% probability of renting from Video Vic's the next time and a 40% chance of renting from MovieMaster. On the other hand, a customer initially renting from MovieMaster has only a 30% likelihood of renting from MovieMaster the next time and a 70% probability of renting from Video Vic's.

a) What is the initial probability vector?

b) What is the transition matrix?

c) What is the probability of a customer renting a movie from each store the second time?

d) What is the probability of a customer renting a movie from each store the third time?

e) What assumption are you making in part d)? How realistic is it?

Solution

a) Initially, each store has 50% of the market, so, the initial probability vector is

$$\begin{array}{rl} & \text{VV} \quad \text{MM} \\ S^{(0)} = & [0.5 \quad 0.5] \end{array}$$

b) The first row of the transition matrix represents the probabilities for the second rental by customers whose initial choice was Video Vic's. There is a 60% chance that the customer returns, so the first entry is 0.6. It is 40% likely that the customer will rent from MovieMaster, so the second entry is 0.4.

Similarly, the second row of the transition matrix represents the probabilities for the second rental by customers whose first choice was MovieMaster. There is a 30% chance that a customer will return on the next visit, and a 70% chance that the customer will try Video Vic's.

$$P = \begin{array}{c} \begin{array}{cc} \text{VV} & \text{MM} \end{array} \\ \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{VV} \\ \text{MM} \end{array}$$

Regardless of which store the customer chooses the first time, you are assuming that there are only two choices for the next visit. Hence, the sum of the probabilities in each row equals one.

c) To find the probabilities of a customer renting from either store on the second visit, calculate the first-step probability vector, $S^{(1)}$:

$$\begin{aligned} S^{(1)} &= S^{(0)}P \\ &= [0.5 \quad 0.5]\begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix} \\ &= [0.65 \quad 0.35] \end{aligned}$$

This new vector shows that there is a 65% probability that a customer will rent a movie from Video Vic's on the second visit to a video store and a 35% chance that the customer will rent from MovieMaster.

d) To determine the probabilities of which store a customer will pick on the third visit, calculate the second-step probability vector, $S^{(2)}$:

$$\begin{aligned} S^{(2)} &= S^{(1)}P \\ &= [0.65 \quad 0.35]\begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix} \\ &= [0.635 \quad 0.365] \end{aligned}$$

So, on a third visit, a customer is 63.5% likely to rent from Video Vic's and 36.5% likely to rent from MovieMaster.

e) To calculate the second-step probabilities, you assume that the conditional transition probabilities do not change. This assumption might not be realistic since customers who are 70% likely to switch away from MovieMaster may not be as much as 40% likely to switch back, unless they forget why they switched in the first place. In other words, Markov chains have no long-term memory. They recall only the latest state in predicting the next one.

Note that the result in Example 1d) could be calculated in another way.

$$
\begin{aligned}
S^{(2)} &= S^{(1)}P \\
&= (S^{(0)}P)P \\
&= S^{(0)}(PP) \qquad \textit{since matrix multiplication is associative} \\
&= S^{(0)}P^2
\end{aligned}
$$

Similarly, $S^{(3)} = S^{(0)}P^3$, and so on. In general, the nth-step probability vector, $S^{(n)}$, is given by

$$S^{(n)} = S^{(0)}P^n$$

This result enables you to determine higher-state probability vectors easily using a graphing calculator or software.

Example 2 Long-Term Market Share

A marketing-research firm has tracked the sales of three brands of hockey sticks. Each year, on average,

- Player-One keeps 70% of its customers, but loses 20% to Slapshot and 10% to Extreme Styx
- Slapshot keeps 65% of its customers, but loses 10% to Extreme Styx and 25% to Player-One
- Extreme Styx keeps 55% of its customers, but loses 30% to Player-One and 15% to Slapshot

a) What is the transition matrix?

b) Assuming each brand begins with an equal market share, determine the market share of each brand after one, two, and three years.

c) Determine the long-range market share of each brand.

d) What assumption must you make to answer part c)?

Solution 1 Using Pencil and Paper

a) The transition matrix is

$$P = \begin{array}{c} \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} \text{P} & \text{S} & \text{E} \end{array} \\ \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.25 & 0.65 & 0.1 \\ 0.3 & 0.15 & 0.55 \end{bmatrix} \end{array}\begin{array}{c} \\ \text{P} \\ \text{S} \\ \text{E} \end{array}$$

b) Assuming each brand begins with an equal market share, the initial probability vector is

$$S^{(0)} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

To determine the market shares of each brand after one year, compute the first-step probability vector.

$$\begin{aligned} S^{(1)} &= S^{(0)}P \\ &= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.25 & 0.65 & 0.1 \\ 0.3 & 0.15 & 0.55 \end{bmatrix} \\ &= [0.41\overline{6} \quad 0.\overline{3} \quad 0.25] \end{aligned}$$

So, after one year Player-One will have a market share of approximately 42%, Slapshot will have 33%, and Extreme Styx will have 25%.

Similarly, you can predict the market shares after two years using

$$\begin{aligned} S^{(2)} &= S^{(1)}P \\ &= [0.41\overline{6} \quad 0.\overline{3} \quad 0.25]\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.25 & 0.65 & 0.1 \\ 0.3 & 0.15 & 0.55 \end{bmatrix} \\ &= [0.45 \quad 0.3375 \quad 0.2125] \end{aligned}$$

After two years, Player-One will have approximately 45% of the market, Slapshot will have 34%, and Extreme Styx will have 21%.

The probabilities after three years are given by

$$\begin{aligned} S^{(3)} &= S^{(2)}P \\ &= [0.45 \quad 0.3375 \quad 0.2125]\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.25 & 0.65 & 0.1 \\ 0.3 & 0.15 & 0.55 \end{bmatrix} \\ &= [0.463 \quad 0.341 \quad 0.196] \end{aligned}$$

After three years, Player-One will have approximately 46% of the market, Slapshot will have 34%, and Extreme Styx will have 20%.

c) The results from part b) suggest that the relative market shares may be converging to a steady state over a long period of time. You can test this hypothesis by calculating higher-state vectors and checking for stability.

For example,

$$S^{(10)} = S^{(9)}P \qquad\qquad S^{(11)} = S^{(10)}P$$
$$= [0.471 \;\; 0.347 \;\; 0.182] \qquad\qquad = [0.471 \;\; 0.347 \;\; 0.182]$$

The values of $S^{(10)}$ and $S^{(11)}$ are equal. It is easy to verify that they are equal to all higher orders of $S^{(n)}$ as well. The Markov chain has reached a **steady state**. A steady-state vector is a probability vector that remains unchanged when multiplied by the transition matrix. A steady state has been reached if

$$\begin{aligned} S^{(n)} &= S^{(n)}P \\ &= S^{(n+1)} \end{aligned}$$

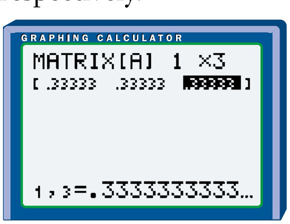
In this case, the steady state vector [0.471 0.347 0.182] indicates that, over a long period of time, Player-One will have approximately 47% of the market for hockey sticks, while Slapshot and Extreme Styx will have 35% and 18%, respectively, based on current trends.

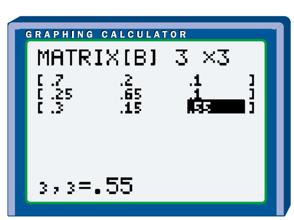
d) The assumption you make in part c) is that the transition matrix does not change, that is, the market trends stay the same over the long term.

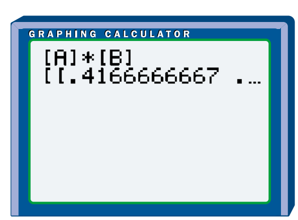
Solution 2 Using a Graphing Calculator

a) Use the MATRX EDIT menu to enter and store a matrix for the transition matrix B.

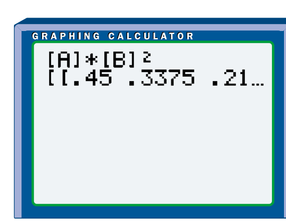
b) Similarly, enter the initial probability vector as matrix A. Then, use the MATRX EDIT menu to enter the calculation $A \times B$ on the home screen. The resulting matrix shows the market shares after one year are 42%, 33%, and 25%, respectively.



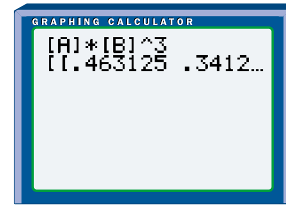




To find the second-step probability vector use the formula $S^{(2)} = S^{(0)}P^2$. Enter $A \times B^2$ using the MATRX NAMES menu and the x^2 key. After two years, therefore, the market shares are 45%, 34%, and 21%, respectively.

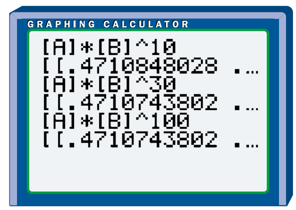


Similarly, enter $A \times B^3$ to find the third-step probability vector. After three years, the market shares are 46%, 34%, and 20%, respectively.



c) Higher-state probability vectors are easy to determine with a graphing calculator.

$$S^{(10)} = S^{(0)}P^{10}$$
$$= [0.471 \ \ 0.347 \ \ 0.182]$$



$$S^{(100)} = S^{(0)}P^{100}$$
$$= [0.471 \ \ 0.347 \ \ 0.182]$$

$S^{(10)}$ and $S^{(100)}$ are equal. The tiny difference between $S^{(10)}$ and $S^{(100)}$ is unimportant since the original data has only two significant digits. Thus, [0.471 0.347 0.182] is a steady-state vector, and the long-term market shares are predicted to be about 47%, 35%, and 18% for Player-One, Slapshot, and Extreme Styx, respectively.

Regular Markov chains always achieve a steady state. A Markov chain is regular if the transition matrix P or some power of P has no zero entries. Thus, regular Markov chains are fairly easy to identify. A regular Markov chain will reach the same steady state *regardless* of the initial probability vector.

Project Prep

In the probability project, you may need to use Markov chains to determine long-term probabilities.

Example 3 Steady State of a Regular Markov Chain

Suppose that Player-One and Slapshot initially split most of the market evenly between them, and that Extreme Styx, a relatively new company, starts with a 10% market share.

a) Determine each company's market share after one year.

b) Predict the long-term market shares.

Solution

a) The initial probability vector is
$$S^{(0)} = [0.45 \ \ 0.45 \ \ 0.1]$$

Using the same transition matrix as in Example 2,

$$S^{(1)} = S^{(0)}P$$
$$= [0.45 \ \ 0.45 \ \ 0.1]\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.25 & 0.65 & 0.1 \\ 0.3 & 0.15 & 0.55 \end{bmatrix}$$
$$= [0.4575 \ \ 0.3975 \ \ 0.145]$$

These market shares differ from those in Example 2, where $S^{(1)} = [0.41\overline{6} \ \ 0.\overline{3} \ \ 0.25]$.

b) $S^{(100)} = S^{(0)}P^{100}$
$= [0.471 \quad 0.347 \quad 0.182]$

In the long term, the steady state is the same as in Example 2. Notice that although the short-term results differ as seen in part a), the same steady state is achieved in the long term.

The steady state of a regular Markov chain can also be determined analytically.

Example 4 Analytic Determination of Steady State

The weather near a certain seaport follows this pattern: If it is a calm day, there is a 70% chance that the next day will be calm and a 30% chance that it will be stormy. If it is a stormy day, the chances are 50/50 that the next day will also be stormy. Determine the long-term probability for the weather at the port.

Solution

The transition matrix for this Markov chain is

$$\begin{array}{cc} & \begin{array}{cc} \text{C} & \text{S} \end{array} \\ P = & \begin{bmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{bmatrix} \begin{array}{c} \text{C} \\ \text{S} \end{array} \end{array}$$

The steady-state vector will be a 1×2 matrix, $S^{(n)} = [p \quad q]$.

The Markov chain will reach a steady state when $S^{(n)} = S^{(n)}P$, so

$$\begin{aligned} [p \quad q] &= [p \quad q] \begin{bmatrix} 0.7 & 0.3 \\ 0.5 & 0.5 \end{bmatrix} \\ &= [0.7p + 0.5q \quad 0.3p + 0.5q] \end{aligned}$$

Setting first elements equal and second elements equal gives two equations in two unknowns. These equations are dependent, so they define only one relationship between p and q.

$$p = 0.7p + 0.5q$$
$$q = 0.3p + 0.5q$$

Subtracting the second equation from the first gives

$$\begin{aligned} p - q &= 0.4p \\ q &= 0.6p \end{aligned}$$

Now, use the fact that the sum of probabilities at any state must equal 1,

$$\begin{aligned} p + q &= 1 \\ p + 0.6p &= 1 \\ p &= \frac{1}{1.6} \\ &= 0.625 \\ q &= 1 - p \\ &= 0.375 \end{aligned}$$

So, the steady-state vector for the weather is $[0.625 \quad 0.375]$. Over the long term, there will be a 62.5% probability of a calm day and 37.5% chance of a stormy day at the seaport.

Key Concepts

- The theory of Markov chains can be applied to probability models in which the outcome of one trial directly affects the outcome of the next trial.
- Regular Markov chains eventually reach a steady state, which can be used to make long-term predictions.

Communicate Your Understanding

1. Why must a transition matrix always be square?

2. Given an initial probability vector $S^{(0)} = [0.4 \quad 0.6]$ and a transition matrix $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}$, state which of the following equations is easier to use for determining the third-step probability vector:
$S^{(3)} = S^{(2)}P$ or $S^{(3)} = S^{(0)}P^3$
Explain your choice.

3. Explain how you can determine whether a Markov chain has reached a steady state after k trials.

4. What property or properties must events A, B, and C have if they are the only possible different states of a Markov chain?

Practise

A

1. Which of the following cannot be an initial probability vector? Explain why.
   a) $[0.2 \quad 0.45 \quad 0.25]$
   b) $[0.29 \quad 0.71]$
   c) $\begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$
   d) $[0.4 \quad -0.1 \quad 0.7]$
   e) $[0.4 \quad 0.2 \quad 0.15 \quad 0.25]$

2. Which of the following cannot be a transition matrix? Explain why.
   a) $\begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.1 & 0 & 0.9 \\ 0.2 & 0.3 & 0.4 \end{bmatrix}$
   b) $\begin{bmatrix} 0.2 & 0.8 \\ 0.65 & 0.35 \end{bmatrix}$
   c) $\begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.3 & 0.22 & 0.48 \end{bmatrix}$

3. Two competing companies, ZapShot and E-pics, manufacture and sell digital cameras. Customer surveys suggest that the companies' market shares can be modelled using a Markov chain with the following initial probability vector $S^{(0)}$ and transition matrix P.

   $S^{(0)} = [0.67 \quad 0.33] \qquad P = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$

   Assume that the first element in the initial probability vector pertains to ZapShot. Explain the significance of
   a) the elements in the initial probability vector
   b) each element of the transition matrix
   c) each element of the product $S^{(0)}P$

Apply, Solve, Communicate

B

4. Refer to question 3.
   a) Which company do you think will increase its long-term market share, based on the information provided? Explain why you think so.
   b) Calculate the steady-state vector for the Markov chain.
   c) Which company increased its market share over the long term?
   d) Compare this result with your answer to part a). Explain any differences.

5. For which of these transition matrices will the Markov chain be regular? In each case, explain why.
   a) $\begin{bmatrix} 0.2 & 0.8 \\ 0.95 & 0.05 \end{bmatrix}$
   b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   c) $\begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.33 & 0.3 & 0.37 \\ 0.5 & 0 & 0.5 \end{bmatrix}$

6. Gina noticed that the performance of her baseball team seemed to depend on the outcome of their previous game. When her team won, there was a 70% chance that they would win the next game. If they lost, however, there was only a 40% chance that they would their next game.
   a) What is the transition matrix of the Markov chain for this situation?
   b) Following a loss, what is the probability that Gina's team will win two games later?
   c) What is the steady-state vector for the Markov chain, and what does it mean?

7. **Application** Two popcorn manufacturers, Ready-Pop and ButterPlus, are competing for the same market. Trends indicate that 65% of consumers who purchase Ready-Pop will stay with Ready-Pop the next time, while 35% will try ButterPlus. Among those who purchase ButterPlus, 75% will buy ButterPlus again and 25% will switch to Ready-Pop. Each popcorn producer initially has 50% of the market.
   - **a)** What is the initial probability vector?
   - **b)** What is the transition matrix?
   - **c)** Determine the first- and second-step probability vectors.
   - **d)** What is the long-term probability that a customer will buy Ready-Pop?

8. **Inquiry/Problem Solving** The weather pattern for a certain region is as follows. On a sunny day, there is a 50% probability that the next day will be sunny, a 30% chance that the next day will be cloudy, and a 20% chance that the next day will be rainy. On a cloudy day, the probability that the next day will be cloudy is 35%, while it is 40% likely to be rainy and 25% likely to be sunny the next day. On a rainy day, there is a 45% chance that it will be rainy the next day, a 20% chance that the next day will be sunny, and a 35% chance that the next day will be cloudy.
   - **a)** What is the transition matrix?
   - **b)** If it is cloudy on Wednesday, what is the probability that it will be sunny on Saturday?
   - **c)** What is the probability that it will be sunny four months from today, according to this model?
   - **d)** What assumptions must you make in part c)? Are they realistic? Why or why not?

9. **Application** On any given day, the stock price for Bluebird Mutual may rise, fall, or remain unchanged. These states, R, F, and U, can be modelled by a Markov chain with the transition matrix:

$$\begin{array}{cc} \begin{array}{ccc} \mathrm{R} & \mathrm{F} & \mathrm{U} \end{array} & \\ \begin{bmatrix} 0.75 & 0.15 & 0.1 \\ 0.25 & 0.6 & 0.15 \\ 0.4 & 0.4 & 0.2 \end{bmatrix} & \begin{array}{c} \mathrm{R} \\ \mathrm{F} \\ \mathrm{U} \end{array} \end{array}$$

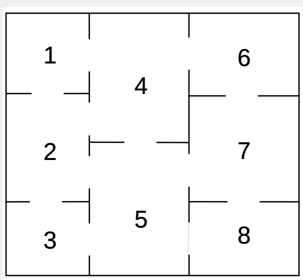
   - **a)** If, after a day of trading, the value of Bluebird's stock has fallen, what is the probability that it will rise the next day?
   - **b)** If Bluebird's value has just risen, what is the likelihood that it will rise one week from now?
   - **c)** Assuming that the behaviour of the Bluebird stock continues to follow this established pattern, would you consider Bluebird to be a safe investment? Explain your answer, and justify your reasoning with appropriate calculations.

10. Assume that each doe produces one female offspring. Let the two states be *D*, a normal doe, and *B*, a doe with bald patches. Determine
   - **a)** the initial probability vector
   - **b)** the transition matrix for each generation of offspring
   - **c)** the long-term probability of a new-born doe developing bald patches
   - **d)** Describe the assumptions which are inherent in this analysis. What other factors could affect the stability of this Markov chain?

ACHIEVEMENT CHECK

Knowledge / Understanding	Thinking / Inquiry / Problem Solving	Communication	Application

11. When Mazemaster, the mouse, is placed in a maze like the one shown below, he will explore the maze by picking the doors at random to move from compartment to compartment. A transition takes place when Mazemaster moves through one of the doors into another compartment. Since all the doors lead to *other* compartments, the probability of moving from a compartment back to the same compartment in a single transition is zero.



a) Construct the transition matrix, P, for the Markov chain.

b) Use technology to calculate P^2, P^3, and P^4.

c) If Mazemaster starts in compartment 1, what is the probability that he will be in compartment 4 after

i) two transitions?

ii) three transitions?

iii) four transitions?

d) Predict where Mazemaster is most likely to be in the long run. Explain the reasoning for your prediction.

e) Calculate the steady-state vector. Does it support your prediction? If not, identify the error in your reasoning in part d).

C

12. **Communication** Refer to Example 4 on page 351.

a) Suppose that the probability of stormy weather on any day following a calm day increases by 0.1. Estimate the effect this change will have on the steady state of the Markov chain. Explain your prediction.

b) Calculate the new steady-state vector and compare the result with your prediction. Discuss any difference between your estimate and the calculated steady state.

c) Repeat parts a) and b) for the situation in which the probability of stormy weather following either a calm or a stormy day increases by 0.1, compared to the data in Example 4.

d) Discuss possible factors that might cause the mathematical model to be altered.

13. For each of the transition matrices below, decide whether the Markov chain is regular and whether it approaches a steady state. (*Hint*: An irregular Markov chain could still have a steady-state vector.)

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \end{bmatrix}$

14. Refer to Example 2 on page 347.

a) Using a graphing calculator, find P^{100}. Describe this matrix.

b) Let $S^{(0)} = [a \;\; b \;\; c]$. Find an expression for the value of $S^{(0)}P^{100}$. Does this expression depend on $S^{(0)}$, P, or both?

c) What property of a regular Markov chain can you deduce from your answer to part b)?

15. **Inquiry/Problem Solving** The transition matrix for a Markov chain with steady-state vector of $\begin{bmatrix} \frac{7}{13} & \frac{6}{13} \end{bmatrix}$ is $\begin{bmatrix} 0.4 & 0.6 \\ m & n \end{bmatrix}$.
Determine the unknown transition matrix elements, m and n.

Career Connection

Investment Broker

Many people use the services of an investment broker to help them invest their earnings. An investment broker provides advice to clients on how to invest their money, based on their individual goals, income, and risk tolerance, among other factors. An investment broker can work for a financial institution, such as a bank or trust company, or a brokerage, which is a company that specializes in investments. An investment broker typically buys, sells, and trades a variety of investment items, including stocks, bonds, mutual funds, and treasury bills.

An investment broker must be able to read and interpret a variety of financial data including periodicals and corporate reports. Based on experience and sound mathematical principles, the successful investment broker must be able to make reasonable predictions of uncertain outcomes.

Because of the nature of this industry, earnings often depend directly on performance. An investment broker typically earns a commission, similar to that for a sales representative. In the short term, the investment broker can expect some fluctuations in earnings. In the long term, strong performers can expect a very comfortable living, while weak performers are not likely to last long in the field.

Usually, an investment broker requires a minimum of a bachelor's degree in economics or business, although related work experience in investments or sales is sometimes an acceptable substitute. A broker must have a licence from the provincial securities commission and must pass specialized courses in order to trade in specific investment products such as securities, options, and futures contracts. The chartered financial analyst (CFA) designation is recommended for brokers wishing to enter the mutual-fund field or other financial-planning services.

WEB CONNECTION

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Visit the above web site and follow the links to find out more about an investment broker and other careers related to mathematics.