

# Review of Key Concepts

## 7.1 Probability Distributions

Refer to the Key Concepts on page 374.

1. Describe the key characteristics of a uniform distribution. Include an example with your description.
2. James has designed a board game that uses a spinner with ten equal sectors numbered 1 to 10. If the spinner stops on an odd number, a player moves forward double that number of squares. However, if the spinner stops on an even number, the player must move back half that number of squares.
  - a) What is the expected move per spin?
  - b) Is this rule “fair”? Explain why you might want an “unfair” spinner rule in a board game.
3. Suppose a lottery has sold 10 000 000 tickets at \$5.00 each. The prizes are as follows:

Prize	Number of Prizes
\$2 000 000	1
\$1 000	500
\$100	10 000
\$5	100 000

Determine the expected value of each ticket.

4. An environmental artist is planning to construct a rectangle with 36 m of fencing as part of an outdoor installation. If the length of the rectangle is a randomly chosen integral number of metres, what is the expected area of this enclosure?
5. Which die has the higher expectation?
  - a) an 8-sided die with its faces numbered 3, 6, 9, and so on, up to 24
  - b) a 12-sided die with its faces numbered 2, 4, 6, and so on, up to 24

## 7.2 Binomial Distributions

Refer to the Key Concepts on page 384.

6. Describe the key characteristics of a binomial distribution. Include an example with your description.
7. Cal’s Coffee prints prize coupons under the rims of 20% of its paper cups. If you buy ten cups of coffee,
  - a) what is the probability that you would win at least seven prizes?
  - b) what is your expected number of prizes?
8. Use a table and a graph to display the probability distribution for the number of times 3 comes up in five rolls of a standard die.
9. A factory produces computer chips with a 0.9% defect rate. In a batch of 100 computer chips, what is the probability that
  - a) only 1 is defective?
  - b) at least 3 are defective?
10. A dart board contains 20 equally-sized sectors numbered 1 to 20. A dart is randomly tossed at the board 10 times.
  - a) What is the probability that the dart lands in the sector labelled 20 a total of 5 times?
  - b) What is the expected number of times the dart would land in a given sector?
11. Each question in a 15-question multiple-choice quiz has 5 possible answers. Suppose you guess randomly at each answer.
  - a) Show the probability distribution for the number of correct answers.
  - b) Verify the formula,  $E(X) = np$ , for the expectation of the number of correct answers.

### 7.3 Geometric Distributions

Refer to the Key Concepts on page 394.

12. Describe the key characteristics of a geometric distribution. Include an example with your description.
13. Your favourite TV station has ten minutes of commercials per hour. What is the expected number of times you could randomly select this channel without hitting a commercial?
14. A factory making printed-circuit boards has a defect rate of 2.4% on one of its production lines. An inspector tests randomly selected circuit boards from this production line.
  - a) What is the probability that the first defective circuit board will be the sixth one tested?
  - b) What is the probability that the first defective circuit board will be among the first six tested?
  - c) What is the expected waiting time until the first defective circuit board?
15. A computer has been programmed to generate a list of random numbers between 1 and 25.
  - a) What is the probability that the number 10 will not appear until the 6th number?
  - b) What is the expected number of trials until a 10 appears?
16. In order to win a particular board game, a player must roll, with two dice, the exact number of spaces remaining to reach the end of the board. Suppose a player is two spaces from the end of the board. Show the probability distribution for the number of rolls required to win, up to ten rolls.

### 7.4 Hypergeometric Distributions

Refer to the Key Concepts on page 403.

17. Describe the key characteristics of a hypergeometric distribution. Include an example with your description.
18. Of the 15 students who solved the challenge question in a mathematics contest, 8 were enrolled in mathematics of data management. Five of the solutions are selected at random for a display.
  - a) Prepare a table and graph of the probability distribution for the number of solutions in the display that were prepared by mathematics of data management students.
  - b) What is the expected number of solutions in the display that were prepared by mathematics of data management students?
19. Seven cards are randomly dealt from a standard deck. Show the probability distribution of the number of cards dealt that are either face cards or aces.
20. One summer, conservation officials caught and tagged 98 beavers in a river's flood plain. Later, 50 beavers were caught and 32 had been tagged. Estimate the size of the beaver population.
21. Suppose that 48 of the tagged beavers in question 20 were males.
  - a) Develop a simulation to estimate the probabilities for the number of tagged males among the 32 beavers captured a second time.
  - b) Verify the results of your simulation with mathematical calculations.