Review of Key Concepts

8.1 Continuous Probability Distributions

Refer to the Key Concepts on page 418.

1. Suppose the commuting time from Georgetown to downtown Toronto varies uniformly from 30 to 55 min, depending on traffic and weather conditions. Construct a graph of this distribution and use the graph to find

   a) the probability that a trip takes 45 min or less

   b) the probability that a trip takes more than 48 min

2. The lifetime of a critical component in microwave ovens is exponentially distributed with $k = 0.16$.

   a) Sketch a graph of this distribution. Identify the distribution by name.

   b) Calculate the approximate probability that this critical component will require replacement in less than five years.

3. Many people invest in the stock market by buying stocks recommended in investment newsletters. The table gives the annual returns after one year for 105 stocks recommended in investment newsletters.

   a) Construct a graph of these data.

   b) Describe the shape of the graph. Use terms such as symmetric, skewed, or bimodal.

   c) Calculate the mean and standard deviation for these data.

Return (%)	Number of Stocks
−15	3
−12	1
−9	1
−6	1
−3	2
0	6
+3	10
+6	16
+9	22
+12	18
+15	9
+18	9
+21	4
+24	1
+27	1
+30	1

8.2 Properties of the Normal Distribution

Refer to the Key Concepts on page 429.

4. An electrician is testing the accuracy of resistors that have a nominal resistance of 15 Ω (ohms). He finds that the distribution of resistances is approximately normal with a mean of 15.08 Ω and a standard deviation of 1.52 Ω. What is the probability that

   a) a resistor selected randomly has a resistance less than 13 Ω?

   b) a resistor selected randomly has a resistance greater than 14.5 Ω?

   c) a resistor selected randomly has a resistance between 13.8 Ω and 16.2 Ω?

5. The results of a blood test at a medical laboratory are normally distributed with $\mu = 60$ and $\sigma = 15$.

   a) What is the probability that a blood test chosen randomly from these data has a score greater than 90?

   b) What percent of these blood tests will have results between 50 and 80?

   c) How low must a score be to lie in the lowest 5% of the results?

6. The lifetimes of a certain brand of photographic light are normally distributed with a mean of 210 h and a standard deviation of 50 h.

   a) What is the probability that a particular light will last more than 250 h?

   b) What percent of lights will need to be replaced within 235 h?

   c) Out of 2000 lights, how many will have a lifetime between 200 h and 400 h?

8.3 Normal Sampling and Modelling

Refer to the Key Concepts on page 438.

7. The list below gives the age in months of 30 deer tagged in an Ontario provincial park last fall.

47	28	31	41	39	25	21	29	26	23
34	25	33	37	28	45	18	36	54	40
33	47	42	29	37	22	42	37	48	64

a) Use a method of your choice to assess whether these data are normally distributed. Explain your conclusion.

b) Find the mean and standard deviation of these data.

c) Determine the probability that a deer selected randomly from this sample is at least 30 months old. State and justify any assumptions you have made.

8. A quality-control inspector chose 20 bolt housings randomly from an assembly line. The interior diameters of these housings are listed in centimetres below.

2.29	2.23	2.48	2.24	2.40	2.23	2.37
2.33	2.37	2.31	2.31	2.26	2.26	2.18
2.33	2.31	2.30	2.30	2.24	2.34	

a) Find the mean and standard deviation for these data.

b) Assume the data are normally distributed. If the minimum acceptable diameter of these bolt housings is 2.25 cm, what proportion of the housings would be rejected as below this minimum?

9. Last year, Satsville High School ran nine classes of mathematics of data management, each with the same number of students. The class-average scores at year-end were as follows:

80.4	70.5	68.9	72.7	83.1	78.6	76.6
74.4	75.8					

a) Find the mean and standard deviation of these class-average scores.

b) How is the standard deviation of the class-average scores related to the underlying distribution of individual scores?

c) A study of the scores for all of the data-management students found that the scores were approximately normally distributed, with a mean of 75.7 and a standard deviation of 24.3. Given that all nine classes were the same size, find the most likely value for this class size. Explain your answer.

10. Use a normal approximation to find the probability that in a given year there will be more than 20 major earthquakes.

Chapter Problem

8.4 Normal Approximation to the Binomial Distribution

Refer to the Key Concepts on page 448.

11. A manufacturer of pencils has 60 dozen pencils randomly chosen from each day's production and checked for defects. A defect rate of 10% is considered acceptable.

   a) Assuming that 10% of all the manufacturer's pencils are actually defective, what is the probability of finding 80 or more defective pencils in this sample?

   b) If 110 pencils are found to be defective in today's sample, is it likely that the manufacturing process needs improvement? Explain your conclusion.

12. A store manager believes that 42% of her customers are repeat business (have visited her store within the last two weeks). Assuming she is correct, what is the probability that out of the next 500 customers, between 200 and 250 customers are repeat business?

8.5 Repeated Sampling and Hypothesis Testing

Refer to the Key Concepts on page 456.

13. An auto body repair shop plans its billing based on an average of 0.9 h to paint a car. The owner recently checked times for the last 50 cars painted and found that the average time for these cars was 1.2 h. She knows that the standard deviation is 0.4 h. Test the significance of this result at $\alpha = 0.10$.

14. A basketball coach claims that the average cost of basketball shoes is less than $80. He surveyed the costs of 36 pairs of basketball shoes in local stores and found the following prices:

$60	$50	$120	$110	$75	$110	$70
$40	$90	$65	$60	$85	$75	$80
$75	$80	$90	$45	$55	$70	$85
$85	$90	$90	$80	$50	$80	$85
$60	$70	$55	$95	$60	$45	$95
$70						

Test the coach's hypothesis at a 5% significance level.

15. A perfume company's long-term market share is estimated to be 6%. After an extensive advertising campaign, 11 out of 150 consumers surveyed claim to have purchased this company's perfume recently. Was the advertising campaign a success? Justify your assessment.

8.6 Confidence Intervals

Refer to the Key Concepts on page 464.

16. A study found that the average time it took for a university graduate to find a job was 5.4 months, with a standard deviation of 0.8 months. If a sample of 64 graduates were surveyed, determine a 95% confidence interval for the mean time to find a job.

17. In a survey of 200 households, 72 had central air-conditioning. Find a 90% confidence interval for the proportion of homes with central air-conditioning.

18. Here are the numbers of employees at 40 selected corporations in southern Ontario.

7685	11 778	11 370	9953	6200
900	2100	1270	1960	887
3100	7300	5400	3114	348
1650	400	873	195	173
725	3472	1570	256	895
120	347	40	2290	4236
850	540	164	285	12
390	60	713	175	213

a) Find a 90% confidence interval for the average number of employees at corporations in southern Ontario.

b) Comment on your findings. Does your confidence interval describe the sample data realistically? What problems exist with constructing a confidence interval for data of this sort?

19. A regional planner has been asked to estimate the average income of businesses in her region. She wants to be 90% confident of her conclusion. She sampled 40 companies and listed their net incomes (in thousands of dollars) as shown below:

84	49	3	133	85	4240	461	60
28	97	14	252	18	16	24	345
254	29	254	5	72	31	23	225
70	8	61	366	77	8	26	10
55	137	158	834	123	47	2	21

Estimate the average income of businesses in this region, with a 90% confidence level.

20. A survey of reading habits found that 63% of those surveyed said they regularly read at least part of a daily newspaper. The results are considered accurate within plus or minus 5%, 19 times in 20. Determine the number of people surveyed.

21. A market-research company found that 14.5% of those surveyed used Gleemodent toothpaste. The company states that the survey is accurate within ±4%, nine times out of ten. How many people did they survey?

22. A city's transportation department surveyed 50 students, 70 city residents, and 30 cyclists concerning their opinion on how well the city supported bicycling as an alternative means of transportation. The table below summarizes the survey results:

Rating	Students	Community	Cyclists
Excellent	0	4	0
Very Good	3	12	0
Good	23	17	8
Not So Good	15	19	8
Poor	7	11	11
Very Poor	2	7	3

a) Construct a 95% confidence interval for the proportion of students who feel that city's support is good or better.

b) Identify three other proportions which you feel are important. Construct confidence intervals for these proportions. Report your findings, together with reasons why these proportions may be important to the city's transportation department.