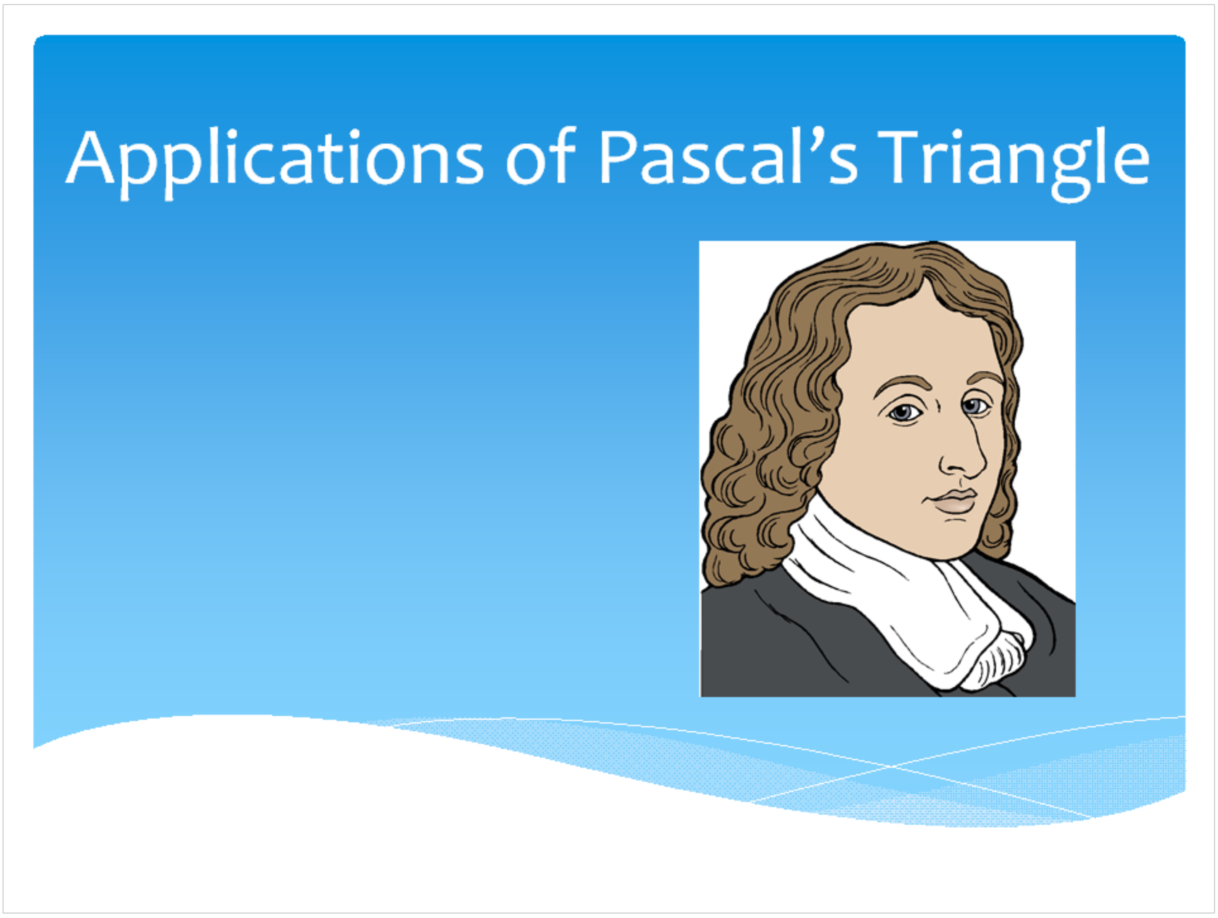
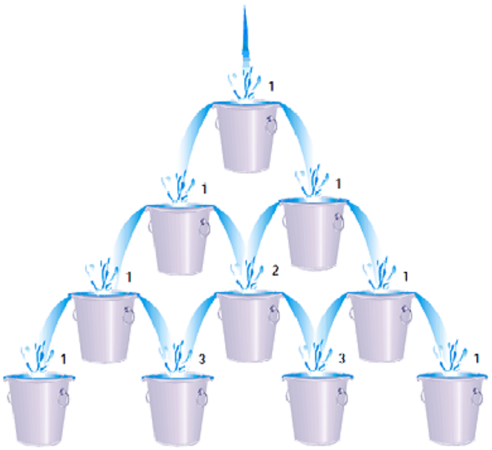

Applications of Pascal’s Triangle

# Applying Pascal's Methods

* The iterative process that generates the terms in Pascal's triangle can also be applied in real life to even the simplest of things such as counting the number of paths or routes between two points.
* You can use Pascal's method to count the different paths that water overflowing from the top bucket could take to each of the buckets in the bottom row.

# Applying Pascal’s Methods




* The water has one path to each of the buckets in the second row. There is one path to each outer bucket of the third row, but two paths to the middle buckets, and so on.
* The number in the diagram match those in Pascal's triangle because they were derived using the same method-Pascal's methods.

***NOTE:**

1. **Pascal's method involves adding two neighboring terms in order to find the term below.**
2. **Pascal's method can be applied to counting paths in a variety of arrays and grids.**

## Example 1: Counting Paths In an Array.

- Determine how many different paths will spell *PASCAL* if you start at the top and proceed to the next row by moving diagonally left or right?

*Note: As you keep branching out, the out extreme values continue to be equal to 1 on both sides (P to C).

```
      P
     A A
    S S S
   C C C C
    A A A
     L L

      1
     1 1
    1 2 1
   1 3 3 1
    4 6 4
    10 10
```

1. Start from P. You can either go to the left A or to the right A. (1 path)
2. There is one path from an A to the left S, two paths from an A to the middle S and one path from an A to the right S.
3. Continuing with this counting reveals that there are 10 different paths leading to each L. Therefore, a total of 20 paths to spell PASCAL. (10+10=20)

## Example 2: Fill in the Missing Numbers

924 792 495 330

1716 1287 825

3003 2112

5115

1. First, consider the row number 1 from the bottom, it is the addition of the previous consecutive 2 terms (3003 and 2112).
2. In row 3, the value 1287(2nd term) is derived from subtracting the term beside it and the term is the next row below it. It can be worked out as follows;
3. (Let the missing term be x.)

# Example 2: Fill in the Missing Numbers

924 …….. 792 …….. 495 330 ……..

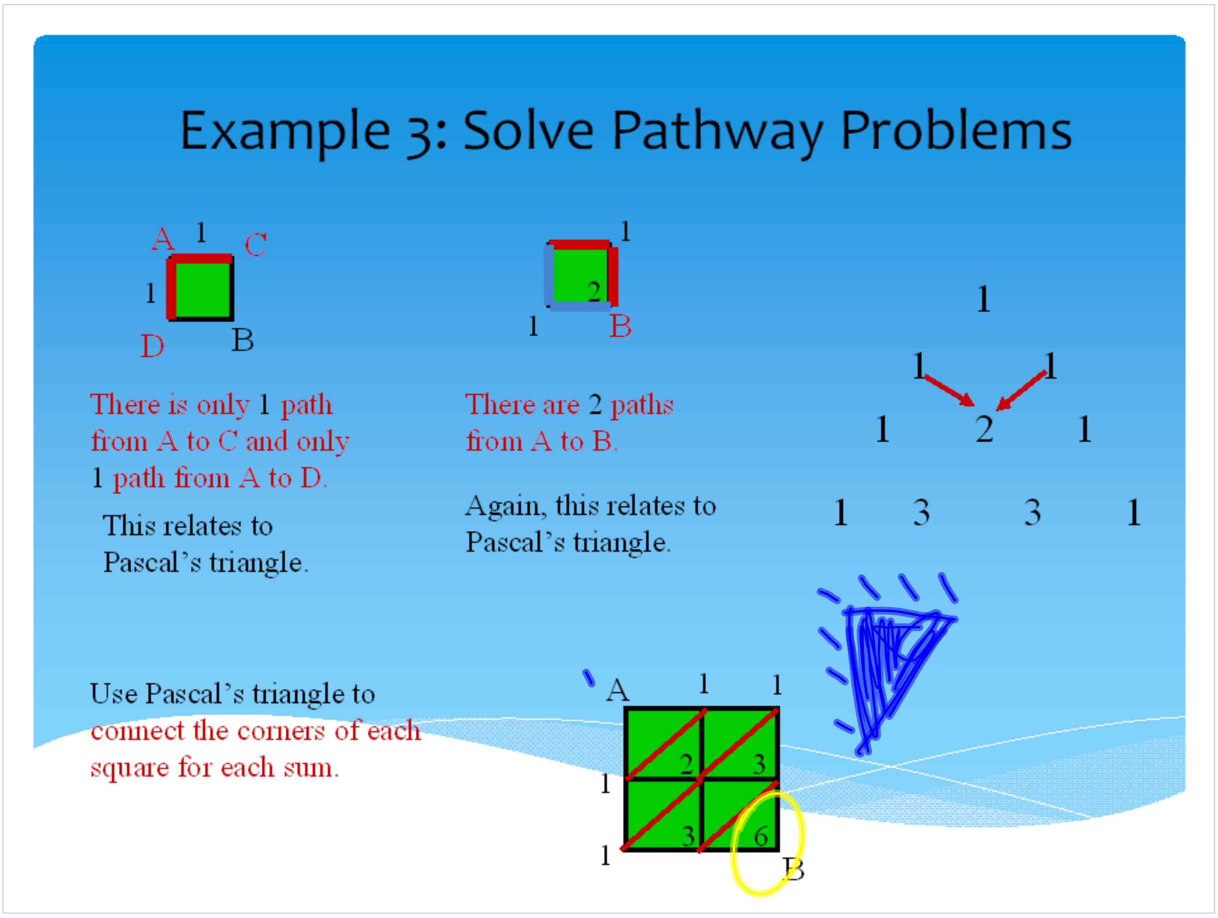
1716 …….. 1287 ……… 825

3003 2112

5115 ……..

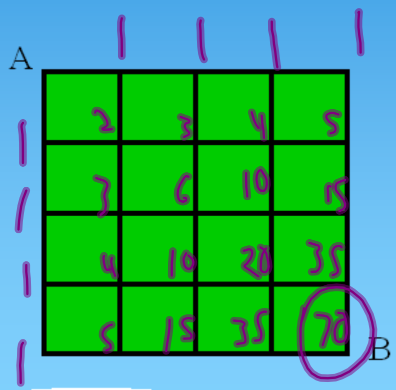
4. According to Pascal's method,
   - X+825=2112
   - X=2112-825
   - X=1287
5. This procedure can be followed and implied to solve the rest of the table

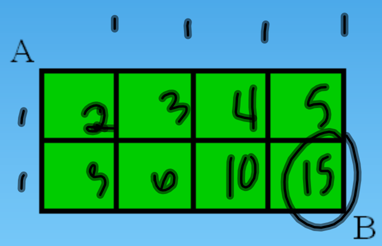
Note: Always begin solving for missing terms in places were there are at least 2 terms, i.e, 2 terms are known (As given in example above).


Example 3: Solve Pathway Problems
A 1 C
1
D B
There is only 1 path from A to C and only 1 path from A to D.
This relates to Pascal's triangle.
1
2
1 B
There are 2 paths from A to B.
Again, this relates to Pascal's triangle.
1
1 1
1 2 1
1 3 3 1
Use Pascal's triangle to connect the corners of each square for each sum.
A 1 1
1 2 3
1 3 6
B

# Example 3: Solve Pathway Problems

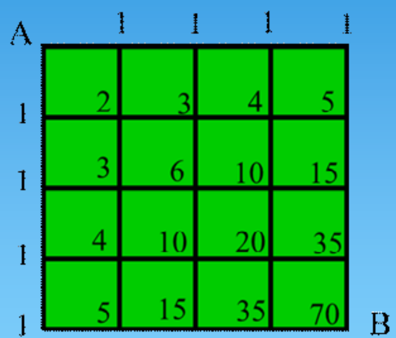
Continue with the pattern of Pascal's triangle to solve larger pathway problems.



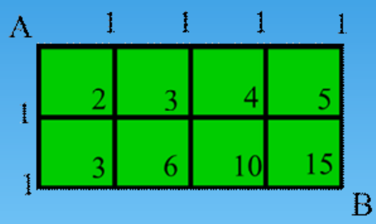



# Example 3: Solve Pathway Problems

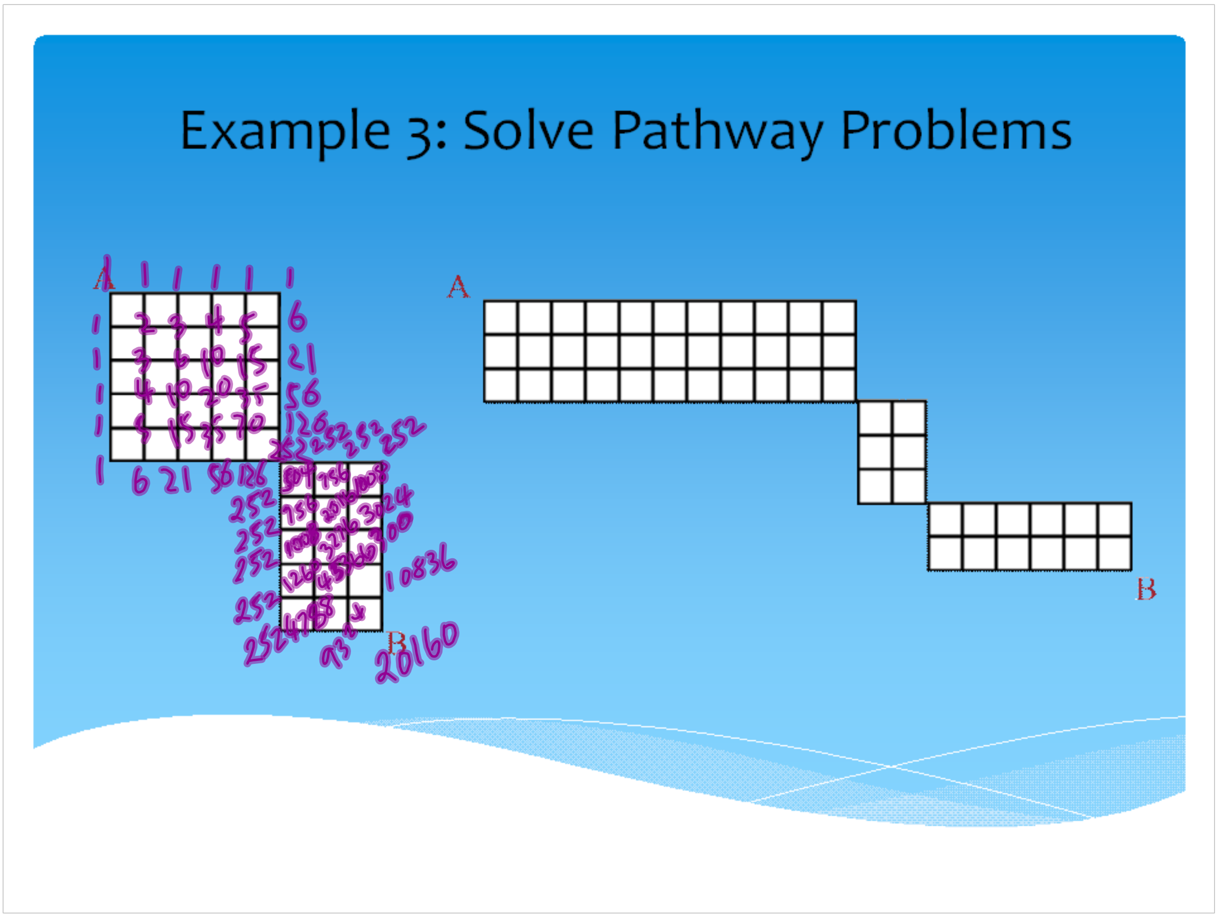
Continue with the pattern of Pascal's triangle to solve larger pathway problems.



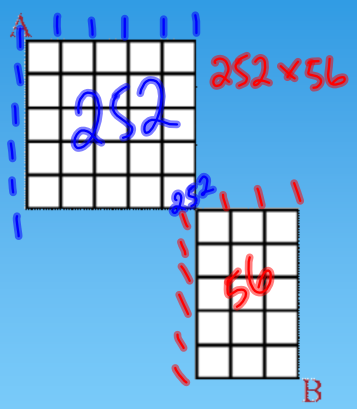

= 70

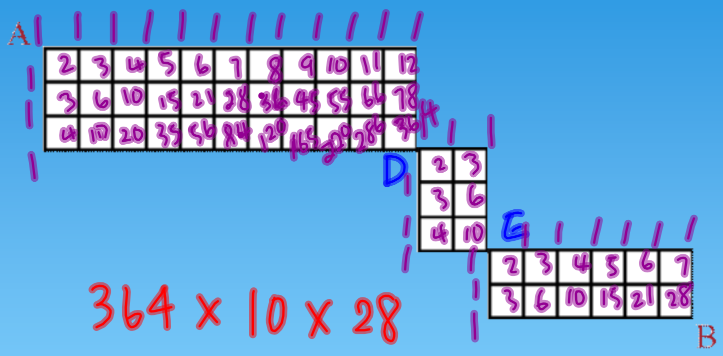



= 15


Example 3: Solve Pathway Problems
A
B
A
B
20160

# Example 3: Solve Pathway Problems



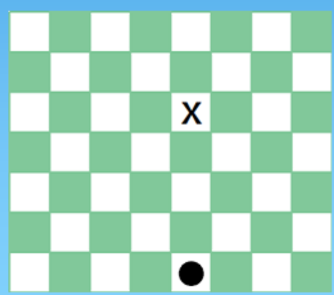




= 14 112

= 101 920

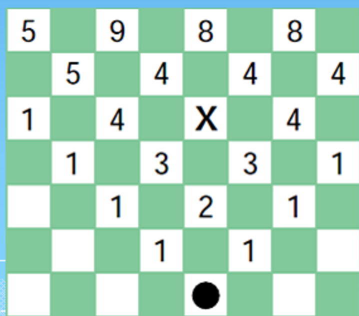
# Example 4: Count Paths on Checkerboard

On the checkerboard shown, the checker can travel only diagonally upward. It cannot move through a square containing an X. Determine the number of paths from the checker's current position to the top of the board.



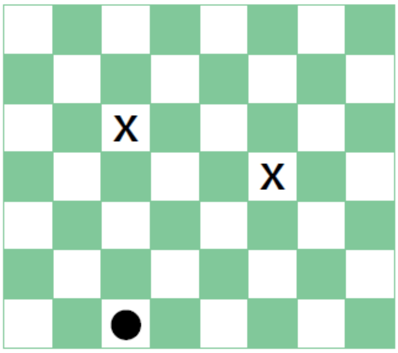
# Example 4: Count Paths on Checkerboard

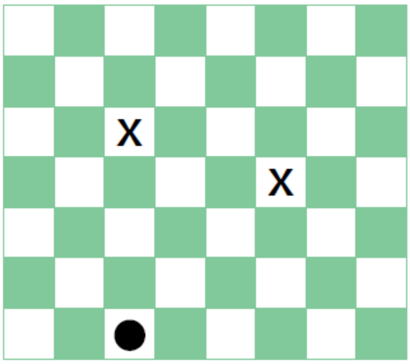
There is one path possible into each of the squares diagonally adjacent to the checker's starting position. From the second row, there are four paths to the third row. Continue this process for the remaining four rows. The square containing an **X** gets a zero or no number since there are no paths through this blocked square.




From left to right, there are 5, 9, 8, and 8 paths to the white squares at the top of the board, making a total of 30 paths.

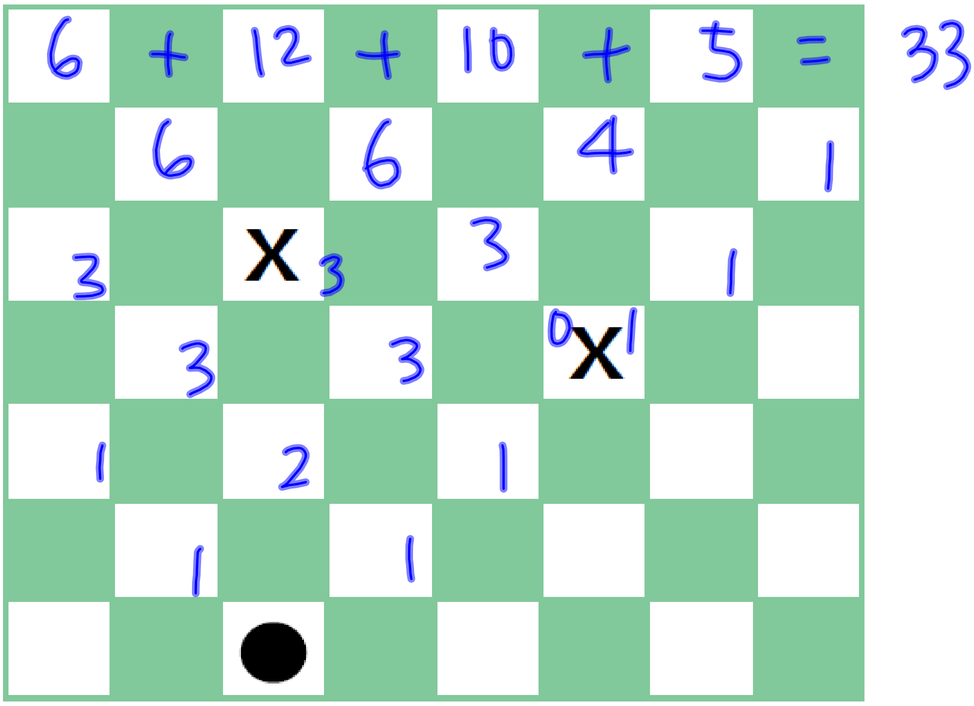
**7.** A checker is placed on a checkerboard as shown. The checker may move diagonally upward. Although it cannot move into a square with an X, the checker may jump over the X into the diagonally opposite square.

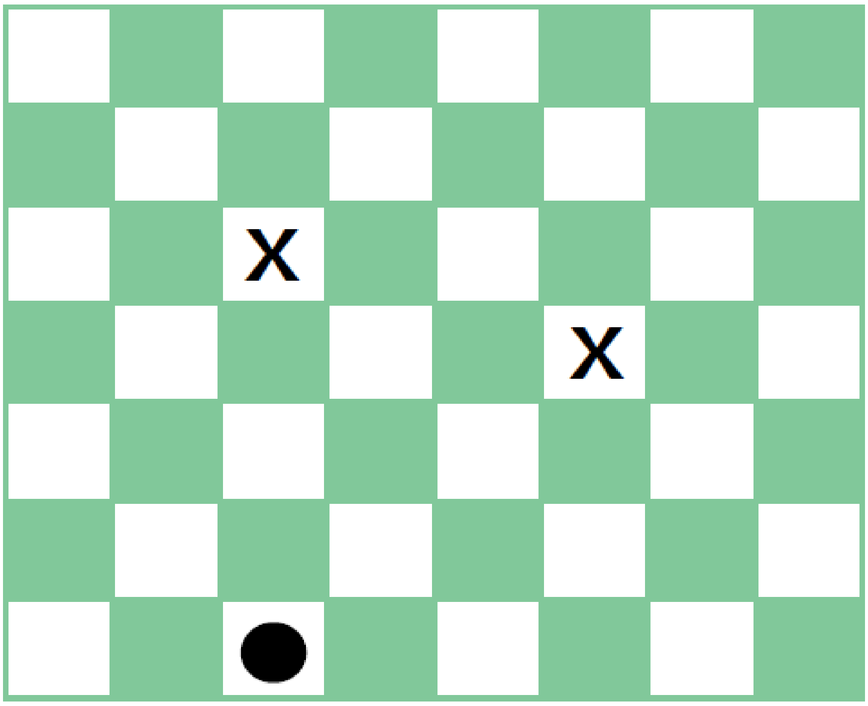





**a)** How many paths are there to the top of the board?

**b)** How many paths would there be if the checker could move both diagonally and straight upward?


6 + 12 + 10 + 5 = 33


X
X

# BINOMIAL EXPANSION

In order to solve binomial terms with high exponents, it is found that Pascal's method is closely related to solving these binomial expressions with higher numbered exponents.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **n = 0** | | | | | | 1 | | | | | | singlet |
| **n = 1** | | | | | 1 | | 1 | | | | | doublet |
| **n = 2** | | | | 1 | | 2 | | 1 | | | | triplet |
| **n = 3** | | | 1 | | 3 | | 3 | | 1 | | | quartet |
| **n = 4** | | 1 | | 4 | | 6 | | 4 | | 1 | | quintet |
| **n = 5** | 1 | | 5 | | 10 | | 10 | | 5 | | 1 | sextet |

# Binomial Expansion

Now consider the following binomial expressions:

- 1
- 1 1
- 1 2 1
- 1 3 3 1

1. $(x + y)^0 = 1$
2. $(x + y)^1 = [(1) X^1] + [(1) Y^1]$
3. $(x + y)^2 = [1 (x)^2] + [2(x)^1(y)^1] + [(1)(y)^2]$
4. $(x + y)^3 = [1(x)^3] + [3(x)^2(y)] + [(3)(x)(y)^2] + [(1)(y)^3]$

## Key points to remember about Binomial expansion using Pascal's method:

- Exponents in EACH term always add up to the highest degree. (Highest degree is referred to as the degree of the polynomial)
- The exponents descend on the 1st variable and ascend on the 2nd variable.
- The coefficients are the terms in the row "n" of Pascal's triangle, where n is the exponent.
- The number of terms while expansion of a binomial = n + 1. (i.e. $(x + y)^{20}$ = 21 terms)