# Binomial Distributions

# Binomial Distributions

- Situations where there are exactly two possible **Mutually Exclusive** outcomes
- For examples:
  - Yes/No to a survey question
  - Product is defective/not defective
  - Correct/Wrong response to a multiple choice question

# Conditions for Binomial Distributions

- There are a fixed number of trials, *n*
- All trials are identical and **independent**
- Each trial has exactly two **Mutually Exclusive** outcomes: Success (S) or Failure (F)
- The probability of success is the same in every trial
- The random variable is the number of successes in a given number of trials

# Notations for Binomial Distributions

- P(S) = p
- P(F) = q = 1 – p
- *n* = number of trials
- *x* = number of successes

* Success is just what is being measured

# Binomial Distributions

Which of the following results in a binomial probability distribution?

- Determining whether each of a 3000 heart pacemakers is acceptable or defective
- Surveying people on their opinions of the current prime minister
- Rolling a die twenty times and counting the number of times a 6 is rolled

# Formulas for Binomial Distributions

Probability in a Binomial Distribution

$$P(x) = {}_nC_x p^x q^{n-x}$$

- where p is the probability of success on any individual and independent trial and
- q = 1 – p is the probability of failure

# Formulas for Binomial Distributions

For ***n*** independent trials, the expectation or

**Expected Value** in a Binomial Distribution

$$E(X) = np$$

- where p is the probability of success on any individual and independent trial and
- n is the number of trials

# Example 1

A report from the Secretary of Health and Human Services stated that 70% of single-vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 10 single-vehicle traffic fatalities that occur at night on a weekend is selected, find the probability that exactly seven involve a driver that is intoxicated.

**This is a binomial experiment where:**

$n = 10$ $x = 7$ $p = 0.70$ $q = 0.30$

# Example 1

A report from the Secretary of Health and Human Services stated that 70% of single-vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 10 single-vehicle traffic fatalities that occur at night on a weekend is selected, find the probability that exactly seven involve a driver that is intoxicated.

**This is a binomial experiment where:**

n = 10 x = 7 p = 0.70 q = 0.30

$$P(7) = {}_{10}C_7 (0.7)^7 (0.3)^3$$

$$P(7) = 0.2668 \text{ or } 0.27$$

# Example 1

A report from the Secretary of Health and Human Services stated that 70% of single-vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 10 single-vehicle traffic fatalities that occur at night on a weekend is selected, find the probability that exactly seven involve a driver that is intoxicated.

What about the probability of **at least 8** involving an intoxicated driver?

# Example 1

What about the probability of **at least 8** involving an intoxicated driver?

$$P(\text{at least } 8) = P(8) \text{ or } P(9) \text{ or } P(10)$$

$$= 0.233474 + 0.121061 + 0.028248$$

$$= 0.382783$$

# Example 2

Tan’s family moves to an area with a different telephone exchange, so they have to get a new telephone number. Telephone numbers in the new exchange start with 466, and all combinations for the four remaining digits are equally likely. Tan’s favourite numbers are the prime numbers 2, 3, 5, and 7.

Find the probability distribution for the number of these prime digits in Tan’s new phone number

# Example 2

The probability of an individual digit being a 2, 3, 5, or 7 is **0.4** (4 out of a choice of 10 digits)

p = 0.4 and q = 0.6

| Number of Primes, $x$ | Probability, $P(x)$ |
|---|---|
| 0 | ${}_4C_0(0.4)^0(0.6)^4 = 0.1296$ |
| 1 | ${}_4C_1(0.4)^1(0.6)^3 = 0.3456$ |
| 2 | ${}_4C_2(0.4)^2(0.6)^2 = 0.3456$ |
| 3 | ${}_4C_3(0.4)^3(0.6)^1 = 0.1536$ |
| 4 | ${}_4C_4(0.4)^4(0.6)^0 = 0.0256$ |

# Example 2

Tan’s family moves to an area with a different telephone exchange, so they have to get a new telephone number. Telephone numbers in the new exchange start with 466, and all combinations for the four remaining digits are equally likely. Tan’s favourite numbers are the prime numbers 2, 3, 5, and 7.

What is the expected number of these prime digits in the new telephone number?

# Example 2

Using the formula for Expected Value

$$\begin{aligned} E(X) &= np \\ &= 4(0.4) \\ &= 1.6 \end{aligned}$$

On average, there will be 1.6 of Tan's favourite digits in telephone numbers in his new exchange.

# Example 2

Using the formula for Expected Value for any Probability Distribution will have the same result:

$$E(X) = x_1 P(X = x_1) + x_2 P(X = x_2) + \ldots + x_n P(X = x_n)$$

$$= \sum_{i=1}^{n} x_i P(X = x_i)$$

$$E(X) = 0(0.1296) + 1(0.3456) + 2(0.3456) + 3(0.1536) + 4(0.0256)$$
$$= 1.6$$

# Example 3

A box of Smarties has different colours in it. There is a 15% chance of getting a blue Smartie.

What is the probability that at least 4 Smarties in a given box are blue out of 10?

What is the expected number of blue candies?

# Example 3

A box of Smarties has different colours in it. There is a 15% chance of getting a blue Smartie.

What is the probability that at least 4 candies in a given box are blue out of 10?

P(at least 4 blue) = 1 – p(0) – p(1) – p(2) – p(3)

= 1 – 0.1969 – 0.3474 – 0.2759 – 0.1298 = 0.05

# Example 3

A box of Smarties has different colours in it.
There is a 15% chance of getting a blue Smartie.

What is the expected number of blue candies?
(i.e. out of the 10)

$E(X) = np = (10)(0.15) = 1.5$