
Combinations

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A **combination** is a selection of a group of objects taken from a larger pool for which the **kinds of objects selected is of importance but not the order** in which they were selected.

How many arrangements are there for the letters ABC?

ABC
ACB
BCA
BAC
CAB
CBA

When the order of the letters is important there are six distinct arrangements or permutations.

However, if order is **not important** and all you wanted was a grouping of ABC, there is only one way, or one combination.

When order matters, you have permutations.
When order does not matter, you have combinations.

The number of combinations of n items taken r at a time is:

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

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When order does not matter, you have combinations.

The number of combinations of n items taken r at a time is:

$${}_nC_r = \frac{n!}{(n-r)!\,r!} = \frac{3!}{0!\;3!}$$

Finding the Number of Combinations

1. Evaluate the following.

a) $\binom{7}{3}$

$${}_7C_3 = \frac{7!}{(7-3)!3!}$$

$= 35$

b) ${}_3C_2 + {}_5C_3$

$${}_3C_2 + {}_5C_3 = \frac{3!}{(3-2)!2!} + \frac{5!}{(5-3)!3!}$$

$= 13$

2. A committee of four students is to be selected from a group of ten students. In how many ways can this be done?

$${}_{10}C_4 = \frac{10!}{(10-4)!\,4!}$$

$= 210$

The committee of four can be selected in 210 ways.

C(7,3)

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= 210

The committee of four can be selected in 210 ways.

Finding the Number of Combinations

3. A company is hiring people to fill five identical positions.

a) There are 12 applicants. How many ways can the five positions be filled?

$$_{12}C_5 = \frac{12!}{(12-5)!\,5!}$$

$$= 792$$

The company can fill the five positions 792 ways.

(The number of combinations of 12 taken 5 at a time is 792).

b) The company wants to hire Applicant A and any four of the others. How many ways can the five positions be filled now?

$$_1C_1 \times {}_{11}C_4 = \frac{1!}{1!\,0!} \times \frac{11!}{7!\,4!}$$

$$= 330$$

With the selection of Applicant A and four others, there are now only 330 ways to fill the positions.

Finding the Number of Combinations

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Finding the Number of Combinations

3. A company is hiring people to fill five identical positions.

c) The president's daughter is among the 12 applicants and must be hired along with Applicant A. How many ways can the positions be filled now?

$$ {}_{2}C_{2} \times {}_{10}C_{3} = \frac{2!}{1!\,2!} \times \frac{10!}{7!3!} $$
$$ = 120 $$

With the selection of Applicant A, the president's daughter, and any 3 others, there are now only 120 ways to fill the positions.

Finding the Number of Combinations

4. There are seven books to choose from.

a) How many ways can five or more books be selected?

Select 5 or 6 or 7:

$$_7C_5 + {}_7C_6 + {}_7C_7 = 21 + 7 + 1$$
$$= 29$$

There are 29 ways to select 5 or more books.

b) If zero to seven books are to be selected, how many ways could this be done?

$$_7C_0 + {}_7C_1 + {}_7C_2 + {}_7C_3 + {}_7C_4 + {}_7C_5 + {}_7C_6 + {}_7C_7 = 128$$

Alternative Strategy:

To find the number of ways of selecting from 0 to n objects, use 2^n. $2^7 = 128$

Finding the Number of Combinations

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Select 5 or 6 or 7: Mutually Exclusive (3 Cases)

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Alternative Strategy:

* To find the number of ways of selecting from 0 to n objects, use 2^n. $2^7 = 128$

Finding the Number of Combinations

5. How many ways can one or more of five different toys be selected?

$$2^n - {}_5C_0 = 2^5 - 1$$
$$= 32 - 1$$
$$= 31$$

There are 31 ways to select 1 to 5 toys from a total of five different toys.

6. There are ten different pictures. How many ways can seven or more be selected?

$${}_{10}C_7 + {}_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10} = 176$$

Finding the Number of Combinations

$_5C_0 = 1$

5. How many ways can one or more of five different toys be selected?

$_5C_1 + {}_5C_2 + \ldots + {}_5C_5$

Indirect Method

$2^n - {}_5C_0 = 2^5 - 1$
$= 32 - 1$
$= 31$

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$_{10}C_7 + {}_{10}C_8 + {}_{10}C_9 + {}_{10}C_{10}$

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Finding the Number of Combinations

7. There are seven women and five men applying for four positions with a company. The hiring committee wants to hire at least one woman. How many different ways can the four positions be filled?

1 woman and 3 men	or, 2 women and 2 men	or, 3 women and 1 man	or, 4 women and 0 men

$$({}_7C_1 \times {}_5C_3) + ({}_7C_2 \times {}_5C_2) + ({}_7C_3 \times {}_5C_1) + ({}_7C_4 \times {}_5C_0) = 490$$

The four positions can be filled 490 different ways.

Alternative Strategy:

Take the total number of combinations and subtract the combinations containing no women:

$${}_{12}C_4 - {}_5C_4 = 490$$

Finding the Number of Combinations

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Alternative Strategy:

Take the total number of combinations and subtract the combinations containing no women:

$${}_{12}C_4 - {}_5C_4 = 490$$

only men

Indirect Method

$${}_{12}C_4 - {}_7C_0 \times {}_5C_4$$

Finding the Number of Combinations

8. How many ways can a girl choose one or more of ten different desserts?

$2^{10} - {}_{10}C_0 = 1023$

9. A math class has 18 male students and 19 female students. A committee of four males and three females is to be selected. How many ways can this be done?

${}_{18}C_4$ x ${}_{19}C_3 = 2\ 965\ 140$

10. A math class has 18 male students and 19 female students. A committee of seven is to be selected. How many ways can this be done, if at least one female student must be selected?

${}_{37}C_7 - {}_{18}C_7$ x ${}_{19}C_0 = 10\ 263$
648

Finding the Number of Combinations

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${}_{37}C_7 - {}_{18}C_7 \times {}_{19}C_0 = 10\,263648$
648

1

Finding the Number of Combinations

11. A committee of six is to be chosen from three girls and seven boys. Two particular boys must be on the committee. Find the number of ways of selecting the committee.

$_{2}C_{2} \times {_{8}C_{4}} = 70$

12. How many five-card hands can be dealt from a standard deck of 52 cards if:

a) each hand must contain two aces?

$_{4}C_{2} \times {_{48}C_{3}} = 103\ 776$

b) each hand must contain three red cards?

$_{26}C_{3} \times {_{26}C_{2}} = 845\ 000$

Finding the Number of Combinations

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Finding the Number of Combinations

13. There are eight points in a coordinate plane and no three points are collinear.

a) How many line segments can be drawn?

$_8C_2 = 28$

b) How many triangles can be drawn?

$_8C_3 = 56$

c) How many quadrilaterals can be drawn?

$_8C_4 = 70$

Finding the Number of Combinations

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Finding the Number of Combinations

14. a) How many diagonals are there in a pentagon?

Pentagon: ${}_5C_2$ - 5 = 5

b) How many diagonals are there in an octagon?

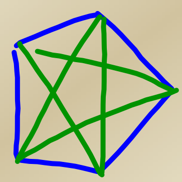
Octagon: ${}_8C_2$ - 8 = 20

c) How many diagonals are there in an *n*-sided polygon?

$$ {}_nC_2 - n = \frac{n!}{(n-2)!\,2!} - n = \frac{n^2 - n - 2n}{2} $$

$$ = \frac{n(n-1)}{2} - \frac{2n}{2} = \frac{n^2 - 3n}{2} $$

Finding the Number of Combinations



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b) How many diagonals are there in an octagon?

Octagon: ${}_8C_2 - 8 = 20$

c) How many diagonals are there in an *n*-sided polygon?

$${}_nC_2 - n = \frac{n!}{(n-2)!\,2!} - n$$

$$= \frac{n(n-1)}{2} - \frac{2n}{2}$$

$$= \frac{n^2 - n - 2n}{2}$$

$$= \frac{n^2 - 3n}{2}$$

Solving Combinations

1. A basketball league has eight teams. Each team must play each other team four times during the season. How many games must be scheduled?

$_8C_2 \times 4 = 112$ 112 games must be scheduled.

2. Solve the equation $_nC_2 = 10$ for n.

$$_nC_2 = 10$$

$$\frac{n!}{(n-2)!2!} = 10$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}2!} = 10$$

$$n^2 - n = 20$$

$$n^2 - n - 20 = 0$$

$$(n - 5)(n + 4) = 0$$

$$n = 5 \text{ or } n = -4$$

Therefore, $n = 5$.

A B C D E F G H

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$$n^2 - n = 20$$

$$n^2 - n - 20 = 0$$

$$(n - 5)(n + 4) = 0$$

$$n = 5 \text{ or } n = -4$$

Reject .

Therefore, $n = 5$.

Solving Combinations

3. a) Show that ${}_{10}C_4 = {}_{10}C_6$.

L.S. | R.S.

$${}_{10}C_4 = {}_{10}C_6$$

$$\frac{10!}{(10-4)!\,4!} = \frac{10!}{(10-6)!\,6!}$$

$$\frac{10!}{6!\,4!} = \frac{10!}{4!\,6!}$$

$$\mathbf{210 = 210}$$

L.S. = R.S.

Therefore, ${}_{10}C_4 = {}_{10}C_6$.

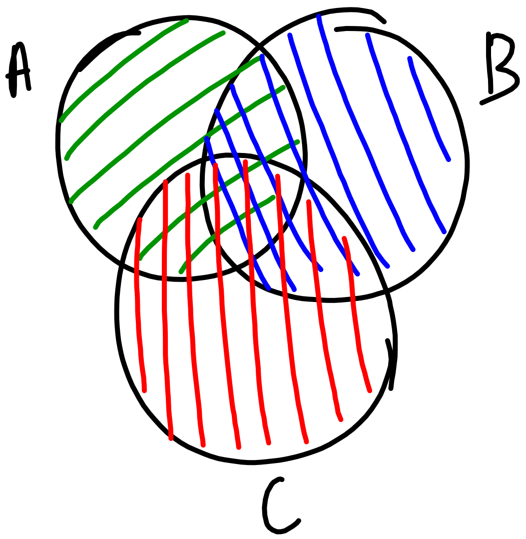
b) Show that ${}_{n}C_r = {}_{n}C_{(n-r)}$.

$$\frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-(n-r))!\,(n-r)!}$$

$$\frac{n!}{(n-r)!\,r!} = \frac{n!}{r!\,(n-r)!}$$

Therefore, ${}_{n}C_r = {}_{n}C_{(n-r)}$.

Solve: ${}_{n}C_5 = {}_{n}C_7$ $\quad n = 12$ $\qquad {}_{14}C_5 = {}_{14}C_n$ $\quad n = 9$



$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$