CONTINUOUS PROBABILITY DISTRIBUTIONS

CONTINUOUS PROBABILITY DISTRIBUTIONS

- Many calculations in nature are of a continuous value, ie. height of a human which require fractions or decimals.
- Unlike a discrete random variable, a ***continuous random variable*** is one that can assume an **uncountable** number of values.
- The graph of a continuous probability distribution is depicted as a smooth curve instead of a bar graph.

POINT PROBABILITIES ARE ZERO

- We cannot list the possible values for a continuous probability distribution because there is an infinite number of them in fractions and decimals
- Since there is an infinite number of values, the probability of each individual value is virtually 0.
- We can determine the probability of a ***range of values*** only.

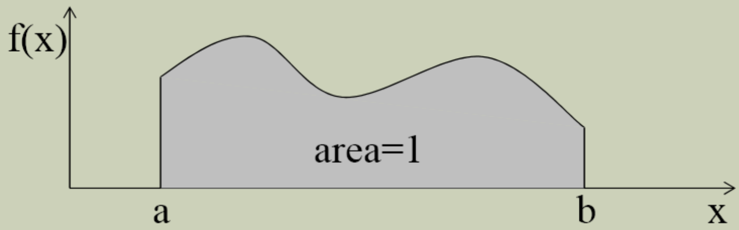
RANGE PROBABILITY ONLY, NOT POINT

- With a **discrete** random variable like tossing a die, it is meaningful to talk about a point probability such as P(X=5).
- In a **continuous** setting (e.g. with time as a random variable), the probability the random variable of interest, say task length, takes exactly 5 minutes is infinitely small, hence P(X=5) = 0.
- *Instead, it is meaningful to range probability such as $P(X \leq 5)$ or $P(2 \leq X \leq 5)$*

PROBABILITY DENSITY FUNCTION

- A function f(x) is called a ***probability density function*** (over the range $a \le x \le b$ if it meets the following requirements:

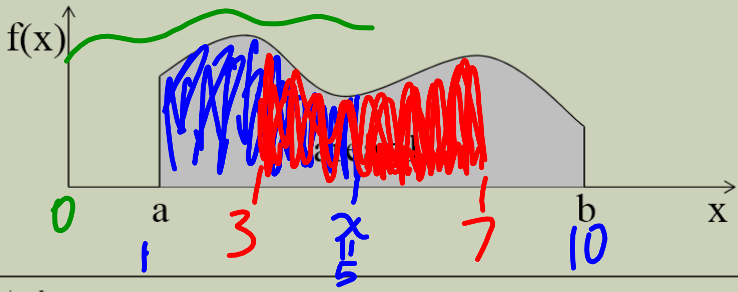
1) $f(x) \ge 0$ for all x between a and b, and
2) The total area under the curve between a and b is 1.0



PROBABILITY DENSITY FUNCTION

- A function f(x) is called a ***probability density function*** (over the range a ≤ x ≤ b if it meets the following requirements:

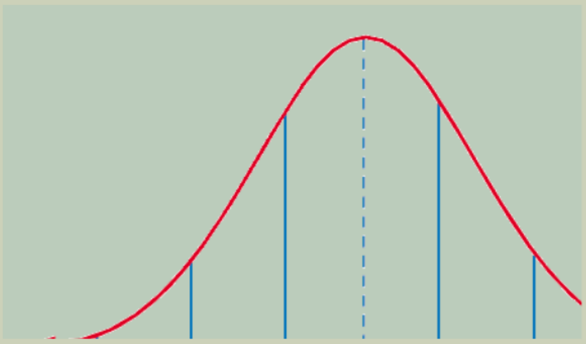
1) f(x) ≥ 0 for all x between a and b, and
2) The total area under the curve between a and b is 1.0



$P(x \leq 5)$ $P(3 \leq x \leq 7)$

GRAPHS OF PROBABILITY DISTRIBUTIONS

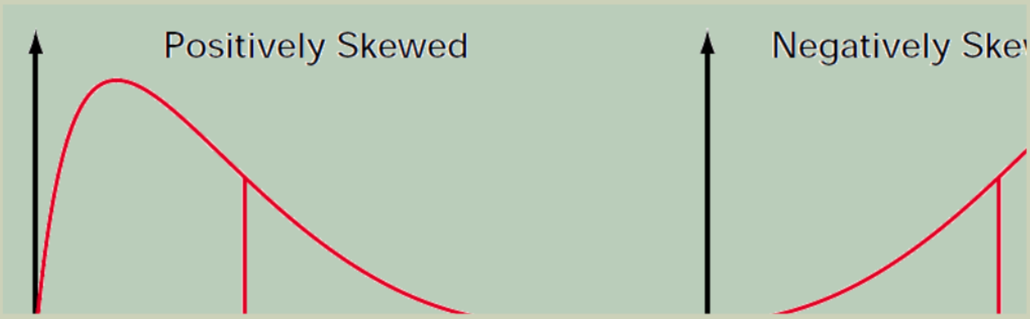
- Symmetrical or Unimodal (The Bell Curve)



- Mean $\doteq$ Mode (Ch.1 → $\doteq$ Median)
- Test Score … hopefully

GRAPHS OF PROBABILITY DISTRIBUTIONS

- Skewed



- Number of Children in a Canadian Family
- Retirement Age
- Amount of Annual Income

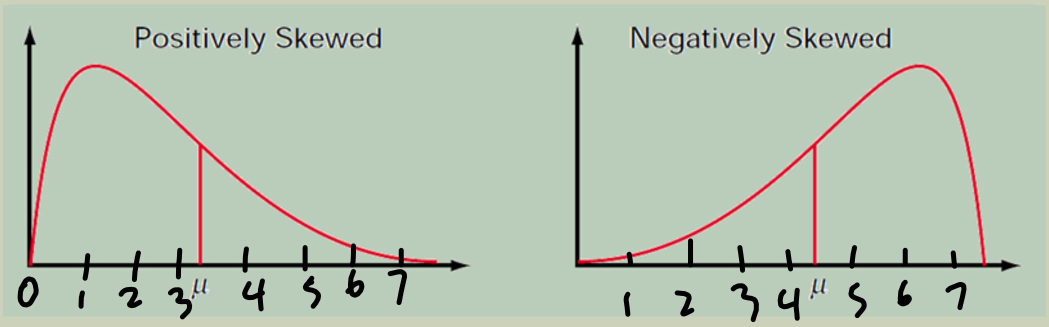
GRAPHS OF PROBABILITY DISTRIBUTIONS

- Skewed

- Number of Children in a Canadian Family +ve
- Retirement Age -ve
- Amount of Annual Income +ve

GRAPHS OF PROBABILITY DISTRIBUTIONS

- Skewed

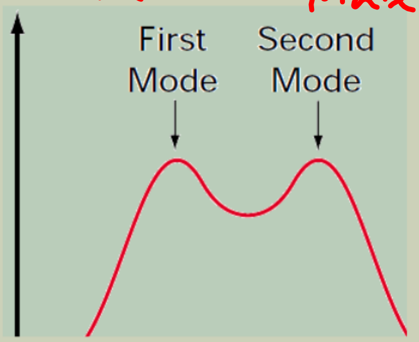


- Number of Children in a Canadian Family +ve
- Retirement Age -ve
- Amount of Annual Income +ve

GRAPHS OF PROBABILITY DISTRIBUTIONS

- Bimodal

Female Male

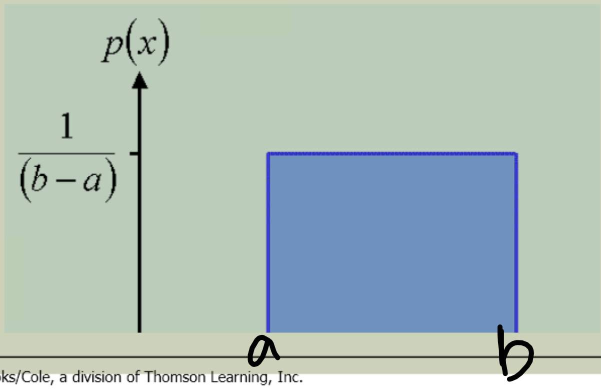


- Adult Shoe Sizes

UNIFORM DISTRIBUTION

- Or the *rectangular probability distribution*
- It is described by the function:

$$f(x) = \frac{1}{b-a}, \quad \text{where } a \le x \le b$$



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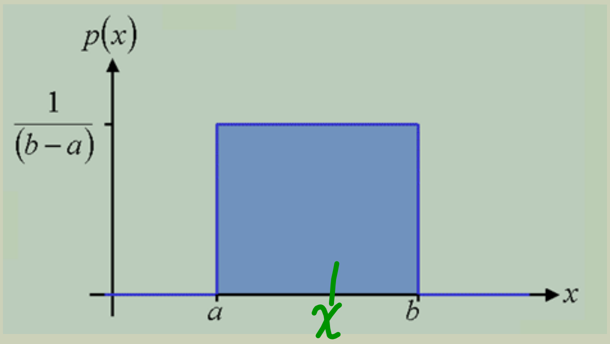
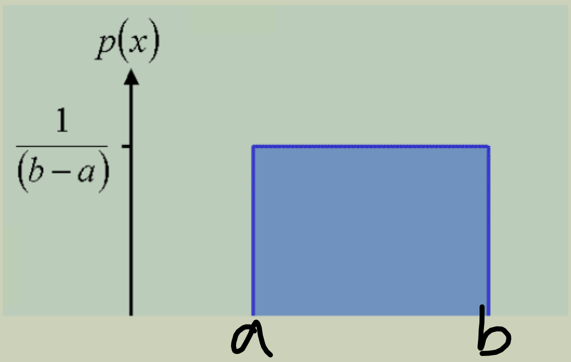


UNIFORM DISTRIBUTION

- The driving time between Toronto and North Bay is found to range evenly between 195 a and 240 b min. What is the probability that the drive will take less than 210 min?



8.10

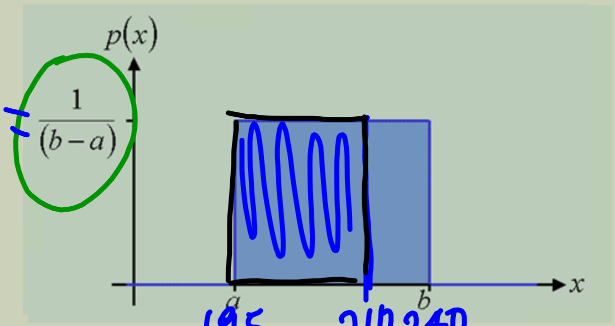
UNIFORM DISTRIBUTION

- The driving time between Toronto and North Bay is found to range evenly between 195 and 240 min. What is the probability that the drive will take less than 210 min?

b a

$$\frac{1}{240-195} = 0.022$$

$0.022 =$



195 210 240

$\text{Area} = 0.022 \times (210 - 195)$

$= 0.33$ Answer

8.10

Why $P(x<210) = P(x \le 210)$?

$P(x=210) = ? = 0$

point probability

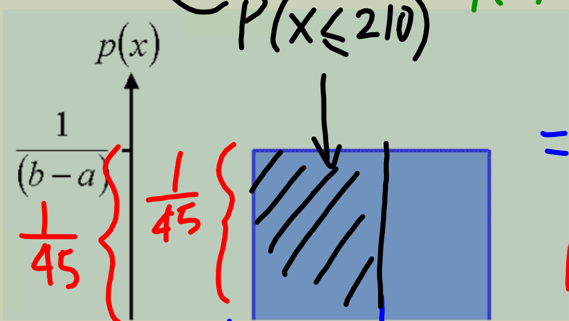
UNIFORM DISTRIBUTION

- The driving time between Toronto and North Bay is found to range evenly between 195 and 240 min. What is the probability that the drive will take less than 210 min?

$P(x \le 210)$

$f(x) = \frac{1}{(b-a)}$

$\frac{1}{45} = \frac{1}{240-195} = \frac{1}{(b-a)} = \frac{1}{240-195}$



$\frac{1}{45}$ $\frac{1}{45}$

195 210 240

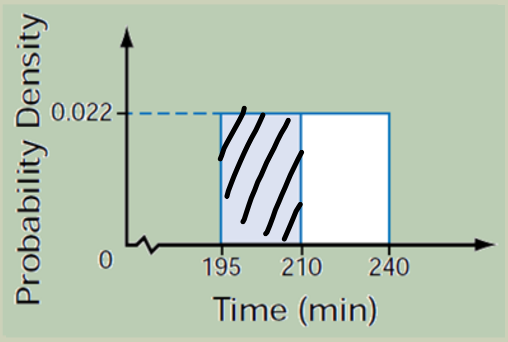
15

$P(x<210) = \frac{1}{45}(210-195)$

$= 0.33$

UNIFORM DISTRIBUTION

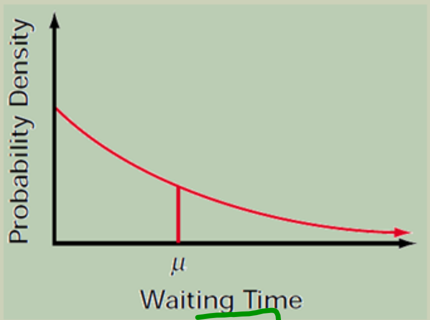
- The driving time between Toronto and North Bay is found to range evenly between 195 and 240 min. What is the probability that the drive will take less than 210 min?



0.33

EXPONENTIAL DISTRIBUTION

- The exponential distribution predicts the waiting times between consecutive events in any random sequence of events.



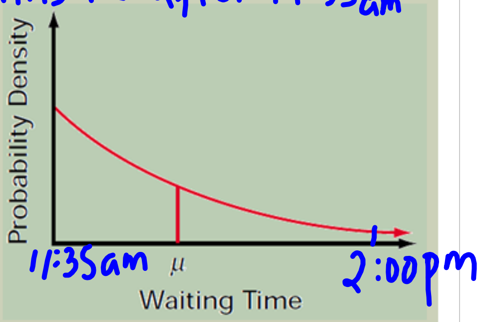
- Probability Density Function: $y = ke^{-kx} \rightarrow \boxed{e^x}$

where $k = \dfrac{1}{\mu}$ is the number of events per unit time and $e \doteq 2.71828$.
The longer the average wait, the smaller the value of k, and the more gradually the graph slopes downward.

EXPONENTIAL DISTRIBUTION

The time it takes to buy lunch from Mainsha after 11:35am

- The exponential distribution predicts the waiting times between consecutive events in any random sequence of events.



- Probability Density Function: $y = ke^{-kx}$

μ = mean

where $k = \dfrac{1}{\mu}$ is the number of events per unit time and $e \doteq 2.71828$.

The longer the average wait, the smaller the value of k, and the more gradually the graph slopes downward.

EXPONENTIAL DISTRIBUTION

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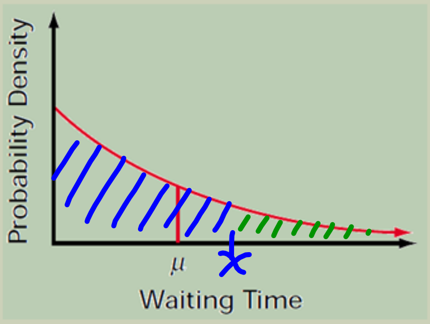

- Cumulative Density Function: $P(X \le x) = 1 - e^{-kx}$

$P(X > x) = 1 - P(X \le x)$

where $k = \dfrac{1}{\mu}$ is the number of events per unit time and $e \doteq 2.71828$.

The longer the average wait, the smaller the value of k, and the more gradually the graph slopes downward.

EXPONENTIAL DISTRIBUTION

- The manager of the local credit union knows that the average length of time it takes to serve one client is 3.5 min and he observes that the length of time to serve a customer has an exponential distribution.
- What is the probability that the next customer will be served in less that 3 minutes?
- What is the probability that the time to serve the next customer will be greater than 5 min?

EXPONENTIAL DISTRIBUTION

- The manager of the local credit union knows that the average length of time it takes to serve one client is 3.5 min and he observes that the length of time to serve a customer has an exponential distribution.

$P(X<3)$

$k=\frac{1}{\mu}$

$=\frac{1}{3.5}=0.29$

- What is the probability that the next customer will be served in less that 3 minutes?
- What is the probability that the time to serve the next customer will be greater than 5 min?

EXPONENTIAL DISTRIBUTION

- What is the probability that the next customer will be served in less that 3 minutes?

$$k = \frac{1}{\mu} = \frac{1}{3.5}$$

The equation for the distribution is $y = 0.29e^{-0.29x}$.

$y = 0.29e^{-0.29x}$ | $y = ke^{-kx}$

An estimate that the waiting time is less than 3 min can be made by adding the probabilities for 0.5, 1.5, and 2.5 min.

x	*p(x)*
0.5	0.250 856
1.5	0.187 707
2.5	0.140 454

The probability is about 0.579 or 0.6.

EXPONENTIAL DISTRIBUTION : By Estimation

- What is the probability that the next customer will be served in less that 3 minutes?

e^x

$k = \frac{1}{\mu}$

$= \frac{1}{3.5}$ $= 0.29$

The equation for the distribution is $\boxed{y = 0.29\boxed{e^{-0.29x}}}$.

$y = ke^{-kx}$

$e^{(-0.29 \times 0.5)}$ $\times 0.29$

An estimate that the waiting time is less than 3 min can be made by adding the probabilities for 0.5, 1.5, and 2.5 min.

x	p(x)
0.5	0.250 856
1.5	0.187 707
2.5	0.140 454

Less than 3 min

The probability is about 0.579 or 0.6.

EXPONENTIAL DISTRIBUTION – By Formula

- What is the probability that the next customer will be served in less than 3 minutes? $P(X \leq x) = 1 - e^{-kx}$

  $\cancel{P(x=3)} = 0$

- Since point probabilities are 0 for continuous probability distributions, $P(X \leq 3) = P(X < 3)$.

  $P(x<3) = P(x \leq 3$ $\because P(x=3)=0$

$$P(x < 3) = P(x \leq 3) = 1 - e^{-0.29(3)}$$
$$= 0.581$$
$$= 0.58$$

The probability is 0.58.

EXPONENTIAL DISTRIBUTION : By Estimation

- What is the probability that the time to serve the next customer will be greater than 5 min?

The probability that the waiting time will be greater than 5 min can be calculated by subtracting the probability that the waiting time will be less than 5 min from 1.

$P(x>5) = 1 - P(x \le 5)$

x	*p(x)*
0.5	0.250856
1.5	0.187707
2.5	0.140454
3.5	0.105097
4.5	0.07864
Estimate	0.762754

$$P(x > 5) = 1 - P(x \le 5)$$
$$= 1 - 0.76$$
$$= 0.24$$

The probability is about 0.24.

$P(x \le 5) = 1 - e^{-0.29(5)}$

EXPONENTIAL DISTRIBUTION – By Formula

Indirect Method

- What is the probability that the time to serve the next customer will be greater than 5 min?

The probability that the waiting time will be greater than 5 min can be calculated by subtracting the probability that the waiting time will be less than 5 min from 1.

$$
\begin{aligned}
P(X > 5) &= 1 - P(X \leq 5) \\
&= 1 - [1 - e^{-0.29(5)}] \quad \text{or} \quad e^{-0.29(5)} \\
&= 1 - 0.765 \\
&= 0.235 \\
&= 0.24
\end{aligned}
$$

$= 0.23$

The probability is 0.24.