MDM4U Factorials and Permutations Date: __________

1. How many ways can we arrange 10 books on a shelf? 

_____ _____ _____ _____ _____ _____ _____ _____ _____ _____ =

This is a very long way to write a number. We will use a short notation for this operation from now. It is denoted by factorial.

For any natural number n, $n! = n(n-1)(n-2)(n-3).........(3)(2)(1)$
Note: **0!=1** $50! = 50 \times 49!$

Example 1: Working with factorials

$$0! = 1$$
$$1! = 1$$
$$2! = 1 \cdot 2 = 2$$
$$3! = 1 \cdot 2 \cdot 3 = 6$$
$$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$$
$$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$$
$$6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$$

$\frac{10!}{7!} =$

$\frac{100!}{98!}$

$6 \times 5! =$

$(n+1)!\,n! =$

$\frac{n!}{(n-2)!} =$

$\frac{1}{n!} + \frac{1}{(n+1)!} =$

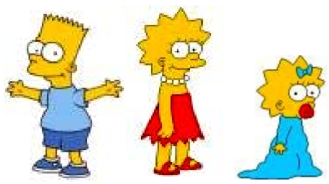
$\frac{n!}{(n-k)!} =$

A **permutation** of all elements of the set of size n is the number of **distinct** arrangements of the elements. It is denoted by ${}_nP_n = n!$ or $P(n,n)$.

Note: A permutation is an arrangement of elements whereby, if an element is selected, it cannot be selected again. In other words, no repeats is allowed

Example 2:

If the Simpsons (Bart, Lisa and Maggie) are to stand in a line for a photograph, how many arrangements could be made?



For each of those ____ choices, there are _____ choices for the second position because the first person cannot be reused.

$\therefore$ There are ______ possible arrangements for these people.

Example 3:

Sandra has a blue, green, red, yellow and purple candy. In how many ways could they be lined up on a table?

A permutation of size ***r*** of ***n*** elements is the number of **distinct** arrangements of the *r* elements.

$${}_nP_r = \frac{n!}{(n-r)!}$$

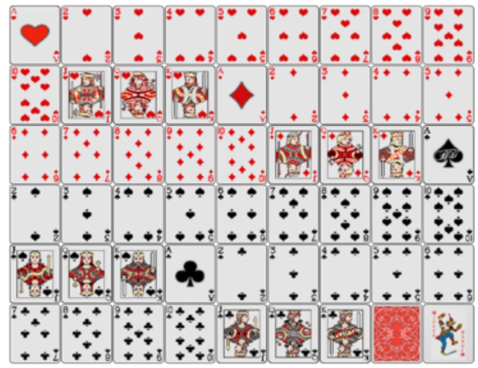
Note: $n > r$ $\quad P(n,r)$

Example 4:

There are 10 magazines in a box. Five of them are to be placed onto a shelf in the library. In how many ways could they be arranged?

Example 5:

From a standard deck of 52 cards, in how many ways could each of the following be arranged?



a) Five face cards (J,Q,K of 4 suits)

b) Eight hearts

c) Nine black cards

Example 6:

In how many ways could the SAC, consisting of a president, vice president, treasurer and publicist be selected from 5 males and 5 females candidate if:

a) There are no restrictions?

b) The president and vice-president may not be of the same sex?

Example 7:

Eric has a briefcase with a three-digit combination lock. He can set the combination himself, and his favourite digits are 5, 6, 7, 8 and 9. Each digit can be used at most once.

a) How many permutations of three of these five digits are there?

b) If you think of each permutation as a three-digit number, how many of these numbers would be odd numbers?

c) How many of the three-digit numbers are even numbers and begin with a 8?

d) How many of the three-digit numbers are even numbers and do not begin with a 8?

e) Is there a connection among the four answers above? If so, state what it is and why it occurs.