GEOMETRIC DISTRIBUTIONS

Geometric Distribution

How many outcomes must occur to get the "first" success?

Geometric distribution is the distribution of the first success in repeated trials of two mutually exclusive outcomes, success or failure.

Geometric Distribution

Conditions for Geometric Distribution:

- All trials are identical and **independent** ✓
- Each trial has exactly two **Mutually Exclusive** outcomes: Success (S) or Failure (F) ✓
- The probability of success is the same in every trial ✓

Geometric Distribution

- P(S) = p
- P(F) = q = 1 − p
- x = number of failures before first success
- $x + 1$ = the trial number of the first success

no fixed number of trials

For the 1st success to occur on the $x + 1^{th}$ trial

- The first x trials must be failures
- the $x + 1^{th}$ trial must be a success

Binomial vs. Geometric

The Binomial Setting	The Geometric Setting
1. Two discrete outcomes: Success/Failure	1. Two discrete outcomes: Success/Failure
2. Probability of success, *p*, is the same for each observation.	2. Probability of success, *p*, is the same for each observation.
3. Observations are **all** independent.	3. Observations are **all** independent.
4. Fixed number of observations, *n*, is the variable of interest	4. Number of trials/failures, *x*, required for 1^{st} success is the variable of interest

Geometric Distribution

Probability in a Geometric Distribution:

$$P(x) = q^{x} p^{1}$$

x+1

↓ failure #s before the 1st success

where *p* is the probability of success in each single trial, *q* is the probability of failure and x is the number of failures (or waiting time for the first success to occur)

Geometric Distribution

Expected Value for Geometric Distribution:

$$E(X) = \sum_{x=0}^{\infty} xP(x) = \frac{q}{p}$$

where *p* is the probability of success in each single trial, and *q* is the probability of failure

Binomial or Geometric

25% of the customers entering a grocery store between 5 p.m. and 7 p.m. use an express checkout. Consider five randomly selected customers, and let X denote the number among the five who use the express checkout.

What is the probability that two customers used the express checkout?

Binomial or Geometric

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Binomial

Binomial or Geometric

Suppose that each of three randomly selected customers purchasing a hot tub at a certain store chooses either an electric (E) or a gas (G) model. Assume that these customers make their choices independently of one another and that 40% of all customers select an electric model.

What is the probability that two customers choose an electric model?

Binomial or Geometric

Suppose that each of three randomly selected customers purchasing a hot tub at a certain store chooses either an electric (E) or a gas (G) model. Assume that these customers make their choices independently of one another and that 40% of all customers select an electric model.

What is the probability that two customers choose an electric model? **Binomial**

Binomial or Geometric

Suppose we have data that suggest that 3% of a company's hard disc drives are defective. You have been asked to determine the probability that the first defective hard drive is the fifth unit tested.

$P(5^{th}) = (0.97)^4(0.03)$

x = # of failures

$x = 4$

Binomial or Geometric

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Geometric

Binomial or Geometric

Miss ⇒ Success

A basketball player makes 80% of her free throws. We put her on the free throw line and ask her to shoot free throws until she misses one. Let X = the number of free throws the player takes until she misses.

What is the probability that she will make 5 shots before she misses? $P(S) = p = 0.2$

$P(5) = (0.8)^5(0.2)$

Binomial or Geometric

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Geometric

Example 1

In a certain town, 30% of adults are very familiar with modern CPR techniques. If adults from this town are randomly selected, what is the probability that the 6th person sampled is the first very familiar with CPR techniques?

success p(s)

$x = 5$

$P(6th) = (0.7)^5(0.3)$

Example 1

In a certain town, 30% of adults are very familiar with modern CPR techniques. If adults from this town are randomly selected, what is the probability that the 6th person sampled is the first very familiar with CPR techniques? $x = 5$

$$P(x=5) = (0.7)^5(0.3) = 0.05$$

Example 2

Suppose that an intersection you pass on the way to school has a traffic light that is green for 40 s and then amber or red for a total of 60 s.

success = green

$P(S) = \frac{40}{100} = 0.4$ $P(F) = 0.6$

What is the probability that the light will be green when you reach the intersection at least once a week?

Example 2

What is the probability that the light will be green when you reach the intersection **at least once** a week?

Out of 100 s, 40 s green & 60 s not green so, **p = 0.4 and q = 0.6**

There are 5 school days in a week.
Consider failures = 0, 1, 2, 3, and 4
to have at least one success

Example 2

What is the probability that the light will be green when you reach the intersection **at least once** a week?

$$P(0 \le x \le 4)$$
$$= (0.6)^0(0.4) + (0.6)^1(0.4) + (0.6)^2(0.4) + (0.6)^3(0.4) + (0.6)^4(0.4)$$
$$= 0.92$$

or use the indirect method

$$P(0 \le x \le 4) = 1 - P(x = 5) = 1 - (0.6)^5(0.4)^0 = 0.92$$

Example 2

Suppose that an intersection you pass on the way to school has a traffic light that is green for 40 s and then amber or red for a total of 60 s.

What is the expected number of days before the light is green when you reach the intersection?

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What is the expected number of days before the light is green when you reach the intersection?

$$E(X) = \frac{0.6}{0.4} = 1.5$$

The expected waiting time before catching a green light is 1.5 days.

Example 3

Jenny has a success rate of 68% for scoring on free throws in basketball. What is the expected waiting time before she misses the basket on a free throw?

$$E(X) = \frac{0.68}{0.32} = 2.12$$

The expectation is that Jenny will score 2.1 on free throws before missing.