

Probability

Mutually Exclusive Events

S = Sample Space

A B

Aces Kings

A♠ A♣
A♥ A♦

K♠ K♣
K♥ K♦

Mutually Exclusive Outcomes

Outcomes are **mutually exclusive** if they cannot happen at the same time.

For example, when you toss a single coin *either* it will land on heads *or* it will land on tails. There are two **mutually exclusive** outcomes.

Outcome A: Head

Outcome B: Tail

When you roll a die *either* it will land on an odd number *or* it will land on an even number. There are two **mutually exclusive** outcomes.

Outcome A: An odd number

Outcome B: An even number



Mutually Exclusive Outcomes

A pupil is chosen at random from the class. Which of the following pairs of outcomes are mutually exclusive?

Outcome A: the pupil has brown eyes.
Outcome B: the pupil has blue eyes.

These outcomes are **mutually exclusive** because a pupil can **either** have brown eyes, blue eyes **or** another colour of eyes.

Outcome C: the pupil has black hair.
Outcome D: the pupil wears glasses.



These outcomes are **not mutually exclusive** because a pupil could have **both** black hair **and** wear glasses.



Adding Mutually Exclusive Outcomes

If two outcomes are mutually exclusive, then:

$$P(A \text{ or } B) = P(A) + P(B)$$

For example, a game is played with the following cards:



What is the probability that a card is a moon or a sun?

$$P(\text{moon}) = \frac{1}{3} \quad \text{and} \quad P(\text{sun}) = \frac{1}{3}$$

Drawing a moon and drawing a sun are mutually exclusive outcomes so,

$$P(\text{moon or sun}) = P(\text{moon}) + P(\text{sun}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$



Adding Mutually Exclusive Outcomes

If two outcomes are mutually exclusive then their probabilities can be added together to find their combined probability.

For example, a game is played with the following cards:



What is the probability that a card is yellow or a star?

$$P(\text{yellow card}) = \frac{1}{3} \quad \text{and} \quad P(\text{star}) = \frac{1}{3}$$

Drawing a yellow card and drawing a star are *not* mutually exclusive outcomes because a card could be yellow and a star.

P (yellow card or star) cannot be found by adding.



Non-Mutually Exclusive Outcomes

If two outcomes are **NOT** mutually exclusive, then
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

For example, a game is played with the following cards:



What is the probability that a card is yellow or a star?

$$P(\text{yellow}) = \frac{1}{3} \quad P(\text{star}) = \frac{1}{3} \quad P(\text{yellow and star}) = \frac{1}{9}$$

Drawing a yellow card and drawing a star are *not* mutually exclusive outcomes because a card could be yellow and a star.

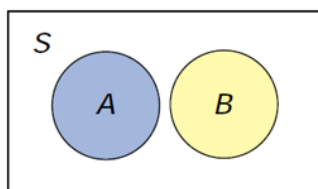
$$P(\text{yellow card or star}) = \frac{1}{3} + \frac{1}{3} - \frac{1}{9} = \frac{5}{9}$$



Mutually vs. Non-Mutually Exclusive

If two events **are** mutually exclusive, then:

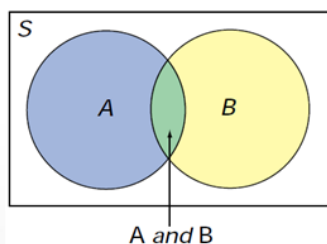
$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B), \text{ but}$$



$$P(A \cap B) = 0$$

If two events are **NOT** mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



↓
To avoid
double-counting



Mutually vs. Non-Mutually Exclusive

If two events **are** mutually exclusive, then:

$$P(A \text{ or } B) = P(A) + P(B)$$

If two events are **NOT** mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Tom has to choose what type of pants he would like to wear. He is most comfortable wearing either khaki or blue jeans. If there are 5 dress pants, 3 blue jeans and 4 khaki pants to choose from, what is the probability that he will receive a pair of pants that he likes?



$$P(\text{jeans or khaki}) = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$



Mutually vs. Non-Mutually Exclusive

If two events **are** mutually exclusive, then:

$$P(A \text{ or } B) = P(A) + P(B)$$

If two events are **NOT** mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Tom has to choose what type of pants he would like to wear. He is most comfortable if he is wearing either kaki or blue jeans. If there are 5 dress pants, 3 blue jeans and 4 kaki pants to choose from, what is the probability that he will receive a pair of pants that he likes?

$$\frac{7}{12}$$



Mutually vs. Non-Mutually Exclusive

If two events **are** mutually exclusive, then:

$$P(A \text{ or } B) = P(A) + P(B)$$

If two events are **NOT** mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

A card is randomly selected from a deck of cards. What is the probability that either a spade or a 7 is selected?

$$P(\text{spade or } 7) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$



Mutually vs. Non-Mutually Exclusive

If two events **are** mutually exclusive, then:

$$P(\mathbf{A \text{ or } B}) = P(\mathbf{A}) + P(\mathbf{B})$$

If two events are **NOT** mutually exclusive, then

$$P(\mathbf{A \text{ or } B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A \text{ and } B})$$

A card is randomly selected from a deck of cards.
What is the probability that either a spade or a 7 is selected?

$$\frac{16}{52} = \frac{4}{13}$$



Sum of All Mutually Exclusive Outcomes

The sum of all mutually exclusive outcomes is 1.

For example, a bag contains red counters, blue counters, yellow counters and green counters.

$$P(\text{blue}) = 0.15 \quad P(\text{yellow}) = 0.4 \quad P(\text{green}) = 0.35$$

What is the probability of drawing a red counter from the bag?

$$P(\text{blue, yellow or green}) = 0.15 + 0.4 + 0.35 = 0.9$$

$$P(\text{red}) = 1 - 0.9 = 0.1$$



Sum of All Mutually Exclusive Outcomes

A box contains bags of chips. The probability of drawing out the following flavours at random are:

$$P(\text{salt and vinegar}) = \frac{2}{5} \quad P(\text{salted}) = \frac{1}{3}$$

The box also contains sour cream and onion chips.

What is the probability of drawing a bag of sour cream and onion chips at random from the box?

$$P(\text{salt and vinegar or salted}) = \frac{2}{5} + \frac{1}{3} = \frac{6+5}{15} = \frac{11}{15}$$

$$P(\text{sour cream and onion}) = 1 - \frac{11}{15} = \frac{4}{15}$$



$$1 - \frac{2}{5} - \frac{1}{3} = \frac{4}{15}$$



Sum of All Mutually Exclusive Outcomes

A box contains bags of chips. The probability of drawing out the following flavours at random are:

$$P(\text{salt and vinegar}) = \frac{2}{5} \quad P(\text{salted}) = \frac{1}{3}$$

The box also contains sour cream and onion chips.

There are 30 bags in the box. How many are there of each flavour?

$$\text{Number of salt and vinegar} = \frac{2}{5} \text{ of } 30 = 12 \text{ packets}$$

$$\text{Number of salted} = \frac{1}{3} \text{ of } 30 = 10 \text{ packets}$$

$$\text{Number of sour cream and onion} = \frac{4}{15} \text{ of } 30 = 8 \text{ packets}$$

