NORMAL APPROXIMATION TO BINOMIAL DISTRIBUTION

BINOMIAL DISTRIBUTION

- Discrete binomial distribution gives the probability of **x** successes in **n** trials

$$P(X = x) = {}_nC_r p^x q^{n-x}$$

- A six-sided die is rolled 60 times. What is the probability that we get "two" on exactly 8 of the rolls? This is easy

$$P(X=8) = {}_{60}C_8 \left(\frac{1}{6}\right)^8 \left(\frac{5}{6}\right)^{52} = 0.12$$

BINOMIAL DISTRIBUTION

- A six-sided die is rolled 60 times. What is the probability that we get "two" on 18 or more of the rolls?

Find $P(X \geq 18)$? $= 1 - P(X < 18)$
$= 1 - P(X=0) - P(X=1) - \ldots P(X=17)$

Where is the Easy button?

NORMAL APPROXIMATION

- The continuous normal distribution can be used to approximate the discrete binomial distribution as long as: (1) $np > 5$
  (2) $nq > 5$
- Only if the above conditions are met, then:

$$\mu = np$$

$$\sigma = \sqrt{npq}$$

NORMAL APPROXIMATION

- A six-sided die is rolled 60 times. What is the probability that we get "two" on exactly 8 of the rolls?

$$P(X=x)={}_nC_r p^x q^{n-x} = 0.12$$

NORMAL APPROXIMATION

- A six-sided die is rolled 60 times. What is the probability that we get "two" on exactly 8 of the rolls?
- Check conditions: (1) $np > 5$ $60(\frac{1}{6}) = 10 > 5$
  (2) $nq > 5$ $60(\frac{5}{6}) = 50 > 5$

$\mu = np = 10$

$\sigma = \sqrt{npq} = 2.89$

$P(x=8) = P(7.5 < X < 8.5)$ Continuity Correction from Disc. to Cont.

$= P\left(\frac{7.5-10}{2.89} < Z < \frac{8.5-10}{2.89}\right)$

$= P(-0.87 < Z < -0.52)$

$P(x<-0.51) - P(x<-0.87)$

$= 0.305 - 0.1922$

$= 0.1128$

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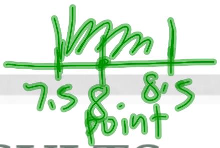
NORMAL APPROXIMATION

- A six-sided die is rolled 60 times. What is the probability that we get "two" on exactly 8 of the rolls?

$\mu = np$

$\sigma = \sqrt{npq}$



COMPARE THE RESULTS

Discrete Binomial	Continuous Normal
$P(x=8) = 0.12$	$P(7.5 < x < 8.5) = 0.1128$

$P(6 \le x \le 8)$

$P(5.5 < x < 8.5)$

EXAMPLE 1

- A bank found that 24% of its loans to new small businesses become delinquent. If 200 small businesses are selected randomly from the bank's files, what is the probability that at least 60 of them are delinquent?

Check if normal approx. is applicable

$np = (200)(0.24) = 48 > 5$ ✓

$nq = (200)(0.76) = 152 > 5$ ✓

$\mu = np = 48 \qquad \sigma = \sqrt{npq} = 6.04$

$$P(X \geq 60) = 1 - P(X < 60)$$
$$= 1 - P(X < 59.5)$$
$$= 1 - P\left(z < \frac{59.5 - 48}{6.04}\right)$$
$$= 1 - P(z < 1.9)$$
$$= 1 - 0.9713$$
$$= 0.0287$$
$$= 2.87\%$$

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EXAMPLE 2

$n = 70 \quad p = 0.42 \quad q = 0.58$

- QuenCola, a soft drink company, knows that it has a 42% market share in one region of the province. QuenCola's marketing department conducts a blind taste test of 70 people at the local mall.

a) What is the probability that fewer than 25 people will choose QuenCola? $P(X < 25)$

b) What is the probability that exactly 25 people will choose QuenCola? $P(X = 25)$

(a) $np = (70)(0.42) = 29.4 > 5$ ✓

$nq = (70)(0.58) = 40.6 > 5$ ✓

$\mu = 29.4$

$\sigma = \sqrt{npq} = 4.13$

$$P(X<25) = P(X<24.5)$$
$$= P\left(Z < \frac{24.5-29.4}{4.13}\right)$$
$$= P(Z<-1.19)$$
$$= 0.1170 = 11.7\%$$

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(b) $P(X=25) = P(24.5 < X < 25.5)$

$$= P(X<25.5) - P(X<24.5)$$
$$= P\left(Z<\frac{25.5-29.4}{4.13}\right) - P\left(Z<\frac{24.5-29.5}{4.13}\right)$$
$$= P(Z<-0.94) - P(Z<-1.19)$$
$$= 0.1736 - 0.117$$
$$= 0.057 = 5.7\%$$

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