The Normal Distribution:

The Normal and Standard Normal

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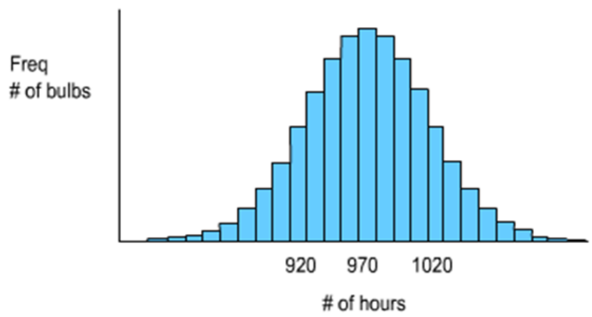
What's a Normal Distribution?

A large number of light bulbs are tested for the number of hours of operation before they burn out. The results show that the mean life of a light bulb is about 970 hours, and the life of most bulbs is close to the mean. A histogram describing light bulb life is shown below.



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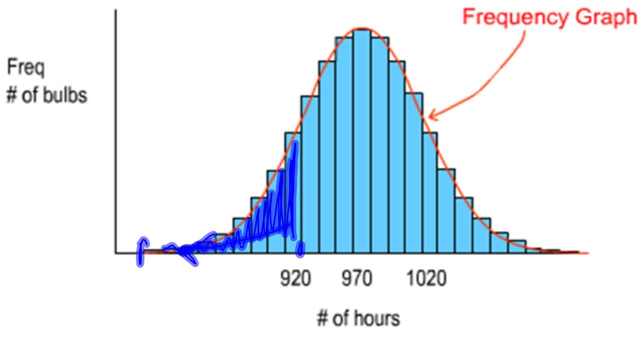
Continuous Random Variable



If we draw a smooth curve through the top of each bar, we get a frequency graph. The area under the curve should be the same as the sum of the areas of the bars in the histogram.

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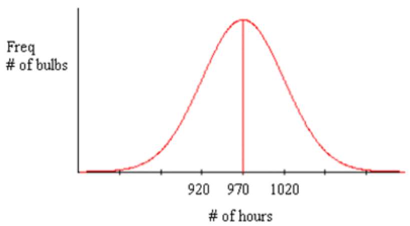
The Normal Distribution

- A symmetrical bell-shaped curve
- The mean, median, and mode are all equal
- 50 % of the data are lower than the mean
- 50 % of the data are higher than the mean



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Examples

There are many situations where data do approximate a normal distribution:

- The heights and weights of adult males in Canada
- The times for athletes to run 5000 metres
- The speed of cars on a busy highway
- The weights of loonies produced at the Royal Mint

All the examples represent continuous data

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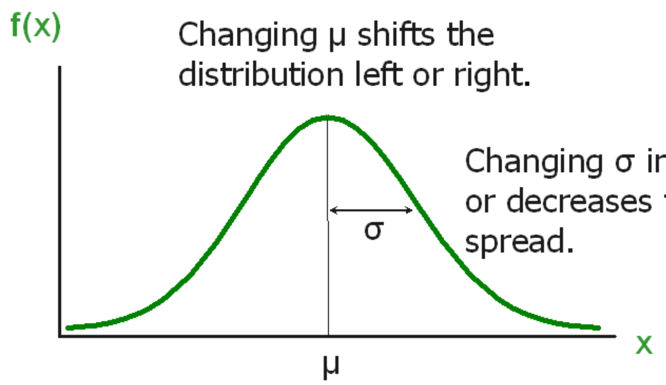
The Normal Distribution



Changing μ shifts the distribution left or right.

Changing σ increases or decreases the spread.

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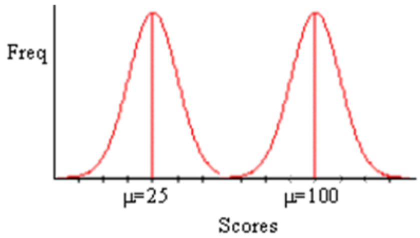
The Normal Distribution Curve

The shape of any normal distribution curve is determined by the mean (μ) and the standard deviation (σ):

- Changing the mean shifts the graph horizontally.



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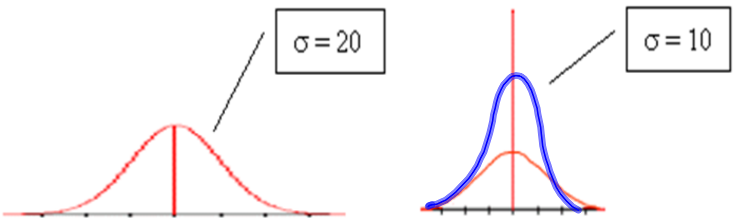
The Normal Distribution Curve

- Changing the standard deviation changes the shape of the curve, making it narrower or wider.



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The Normal Distribution

- Also known as the Gaussian distribution
- The probability density function f(x) for a normal distribution is:

z- score

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

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The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Note constants:
π=3.14159 …
e=2.71828 …

This is a bell shaped curve with different centers and spreads depending on μ and σ

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The Empirical Rule

No matter what μ and σ are:

- Area between $\mu-\sigma$ and $\mu+\sigma$ is about 68%
- Area between $\mu-2\sigma$ and $\mu+2\sigma$ is about 95%
- Area between $\mu-3\sigma$ and $\mu+3\sigma$ is about 99.7%
- Almost all values fall within 3 standard deviations

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Key Areas under the Curve

- Since we know the shape of the curve, we can calculate the area under the curve.
- The percentage of that area can be used to determine the probability that a given value could be pulled from a given distribution

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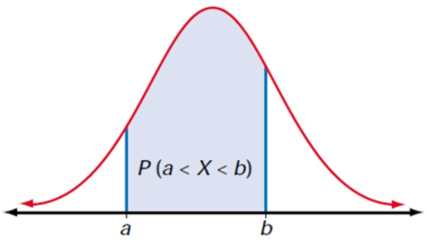
Key Areas under the Curve

- The area under the curve tells us about the probability- in other words we can obtain a p-value for our result (data) by treating it as a normally distributed data set.



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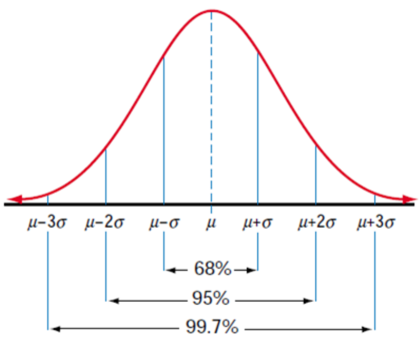
Key Areas under the Curve

For normal distributions :

- ± 1 SD ~ 68%
- ± 2 SD ~ 95%
- ± 3 SD ~ 99.7%



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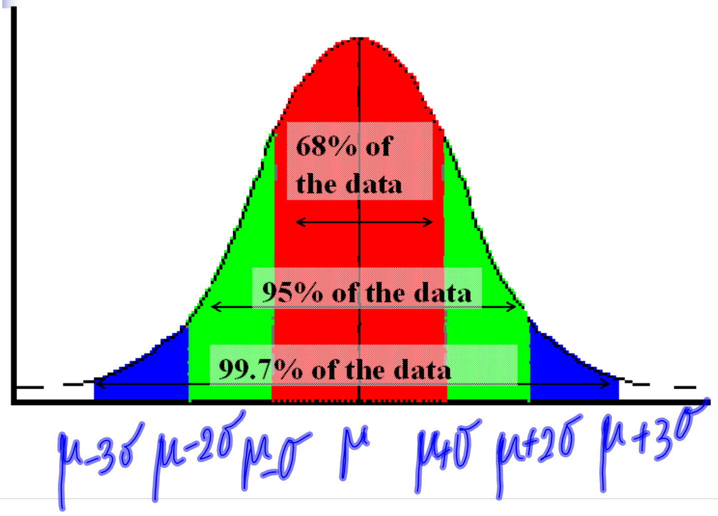
68-95-99.7 Rule



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Applying the Empirical Rule

For example:

The mean of the weight of the women = 127.8

The standard deviation (SD) = 15.5

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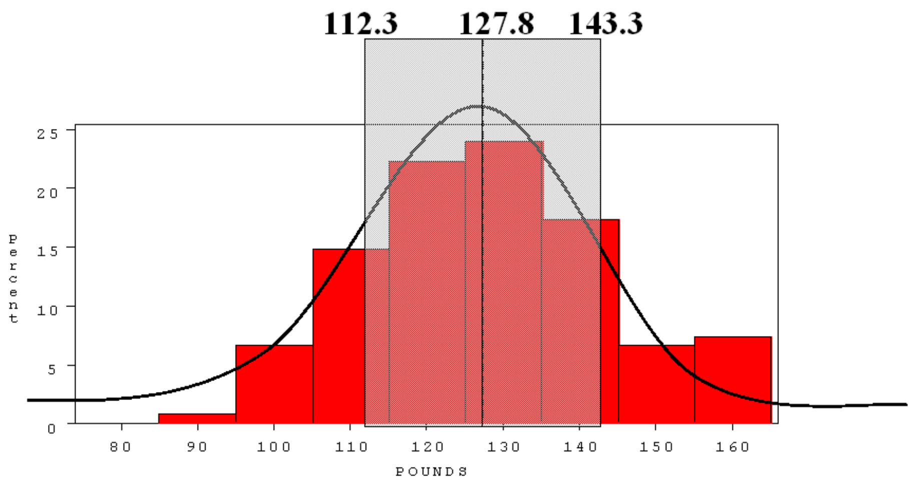
68% of 120 = (0.68)(120) = ~ 82 runners

In fact, 79 runners fall within 1-SD (15.5 lbs) of the mean.



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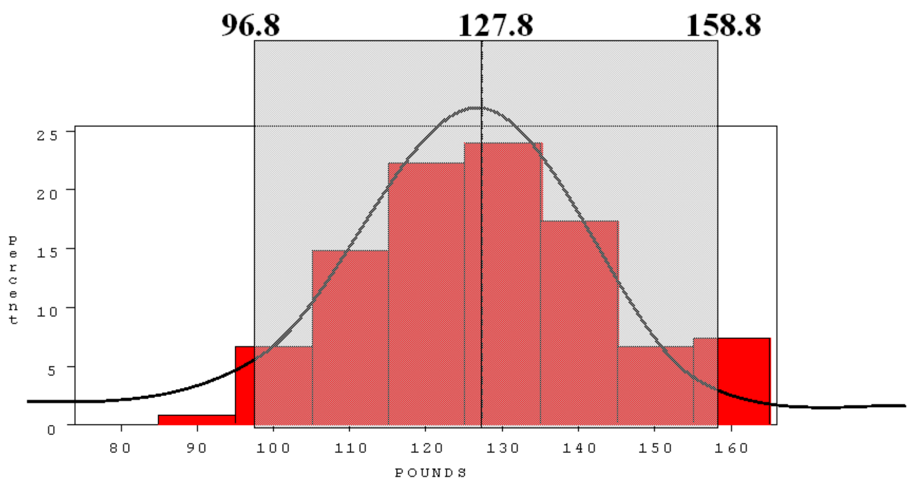
95% of 120 = (0.95)(120) = ~ 114 runners

In fact, 115 runners fall within 2-SD's of the mean.



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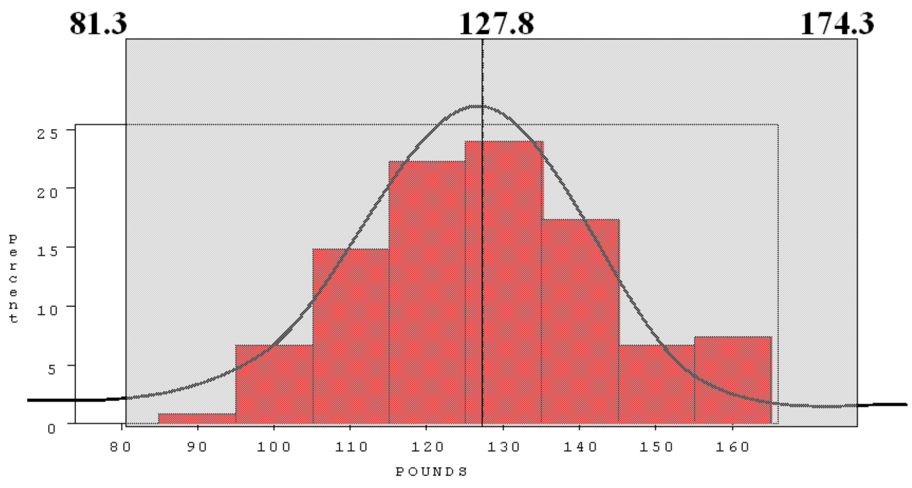
99.7% of 120 = (0.997)(120) = 119.6 runners

In fact, all 120 runners fall within 3-SD's of the mean.



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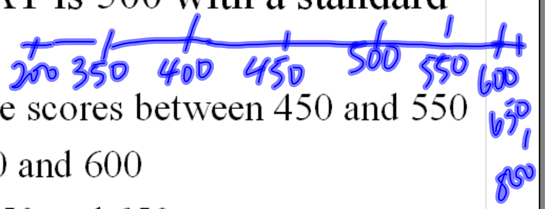
Example

- Suppose SAT scores roughly follows a normal distribution in the U.S. population of college-bound students (with range restricted to 200-800), and the average math SAT is 500 with a standard deviation of 50, then:
  - 68% of students will have scores between 450 and 550
  - 95% will be between 400 and 600
  - 99.7% will be between 350 and 650



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The Problem

- **BUT…**
- What if you wanted to know the math SAT score corresponding to the 90th percentile (=90% of students are lower)?
- It is easier to use the Z-Score and the Standard Normal Curve

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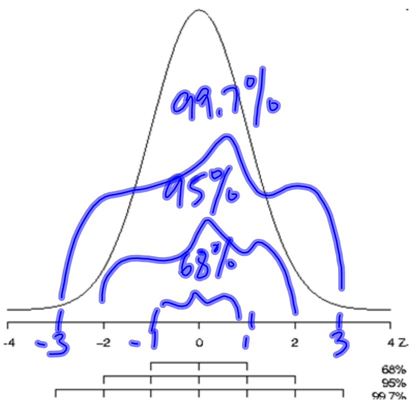
Standard Normal Distribution

- Bell shaped
- μ**=0**
- σ**=1**



- Note: not all bell shaped distributions are standard normal distributions

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The Standard Normal (Z):

The formula for the standardized normal probability density function is ...

$$p(Z) = \frac{1}{(1)\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{Z-0}{1}\right)^2} = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}(Z)^2}$$

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The Standard Normal Distribution (Z)

All normal distributions can be converted into the standard normal curve by subtracting the mean and dividing by the standard deviation:

$$Z = \frac{X - \mu}{\sigma}$$

Then we just have to look up the Z-Score table!

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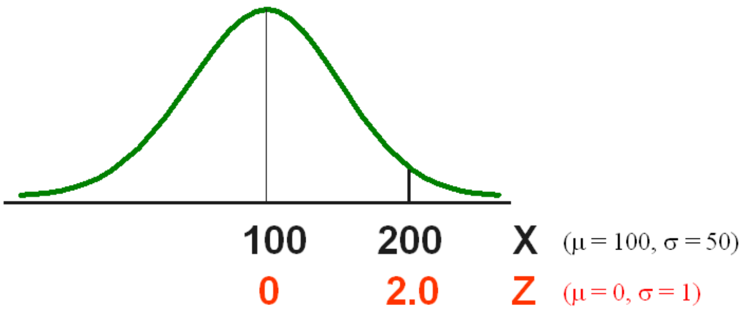
Comparing X and Z units



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Example

For example:

- What's the probability of getting a math SAT score of 575 or less, μ=500 and σ=50?

X

$$Z = \frac{575 - 500}{50} = 1.5$$

$$Z = \frac{X - \mu}{\sigma}$$

Look up Z= 1.5 in standard normal chart

P(Z<575)= .9332

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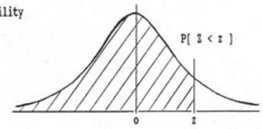
Looking up probabilities in the standard normal table

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Z=1.50

Z=1.50

What is the area to the left of Z=1.50 in a standard normal curve?

0. 9332

Area is 93.32%

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Looking up probabilities in the standard normal table

1.525

= 0.9376

STANDARD STATISTICAL TABLES

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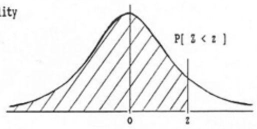

1.52 1.53

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
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1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Z=1.51

Z=1.51

What is the area to the left of Z=1.51 in a standard normal curve?

0.9345

Area is 93.45%

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The Inverse?

- ϕ(area)= Z: gives the Z-value that goes with the probability you want
- For example, recall SAT math scores example. What's the score that corresponds to the 90th percentile? 90%
- In the Table, find the Z-value that corresponds to an area of 90%...

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STANDARD STATISTICAL TABLES

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i.e.

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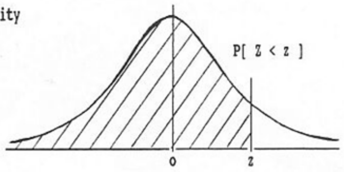

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1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

90% area corresponds to a Z score of about 1.28.

Z (from x)

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The Inverse?

$z = \frac{X-\mu}{\sigma}$

$X = z\sigma + \mu$

Z=1.28; convert back to raw SAT score →

1.28 = X – 500 =1.28 (50)

X=1.28(50) + 500 = 564

(1.28 standard deviations above the mean!)

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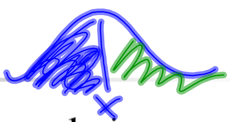
Practice problem



If birth weights in a population are normally distributed with a mean of 109 (μ) oz and a standard deviation of 13 oz, (σ)

a. What is the chance of obtaining a birth weight of 141 oz *or heavier* when sampling birth records at random?

b. What is the chance of obtaining a birth weight of 120 *or lighter*?

$X \to z$

$z = \frac{141-109}{13} = 2.46$

$1 - P(X<141) = 1 - 0.9931 = 0.0069 = 0.69\%$

$P(X<120) = P(z < 0.85) = 0.8023 = 80.23\%$

$z = \frac{120-109}{13} = 0.85$

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Answer

a. What is the chance of obtaining a birth weight of 141 oz *or heavier* when sampling birth records at random?

$$Z = \frac{141 - 109}{13} = 2.46$$

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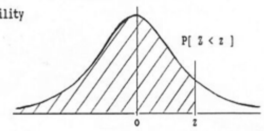

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Area to the left of Z=2.46 is .9931

Area to the right of 2.46 is: 1-.9931 = .0069 or .69%

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Answer

b. What is the chance of obtaining a birth weight of 120 *or lighter*?

$$Z = \frac{120 - 109}{13} = .85$$

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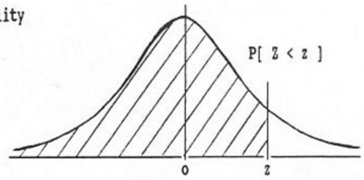
Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z
i.e.

$$P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



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2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

Area to the left of Z=0.85 is .8023 or 80.23%.

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Example 1 **Predictions From a Normal Model**

Giselle is 168 cm tall. In her high school, boys' heights are normally distributed with a mean of 174 cm and a standard deviation of 6 cm. What is the probability that the first boy Giselle meets at school tomorrow will be taller than she is?

$$P(x>168) = 1 - P(x<168)$$
$$= 1 - P(z<-1)$$
$$= 1 - 0.1587$$
$$= 0.8413$$
$$= 84.13\%$$

$x \to z$

$$z = \frac{168-174}{6} = -1$$

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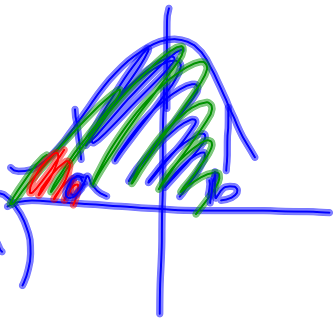
$$P(a<x<b)$$
$$= P(x<b) - P(x<a)$$



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Example 3 Standardized Test Scores

To qualify for a special program at university, Sharma had to write a standardized test. The test had a maximum score of 750, with a mean score of 540 and a standard deviation of 70. Scores on this test were normally distributed. Only those applicants scoring above the third quartile (the top 25%) are admitted to the program. Sharma scored 655 on this test. Will she be admitted to the program?

95th

$Z = \frac{655 - 540}{70}$

$= 1.64$

$P(Z < 1.64) = 0.9495$

greater 0.75

She is in the top 5%.

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Using Normal Dist. to approximate Discrete Dist.

Example 2 Candy Boxes

A company produces boxes of candy-coated chocolate pieces. The number of pieces in each box is assumed to be normally distributed with a mean of 48 pieces and a standard deviation of 4.3 pieces. Quality control will reject any box with fewer than 44 pieces. Boxes with 55 or more pieces will result in excess costs to the company.

Discrete!

a) What is the probability that a box selected at random contains exactly 50 pieces? $P(49.5 < X < 50.5)$

b) What percent of the production will be rejected by quality control as containing too few pieces?

c) Each filling machine produces 130 000 boxes per shift. How many of these will lie within the acceptable range? $P(44 \leq X \leq 54)$ → Discrete ↓ $P(43.5 < X < 54.5)$ continuous.

d) If you owned this company, what conclusions might you reach about your current production process?

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(a) $P(49.5 < X < 50.5)$

$= P(X < 50.5) - P(X < 49.5)$

$= P\left(Z < \frac{50.5-48}{4.3}\right) - P\left(Z < \frac{49.5-48}{4.3}\right)$

$= P(Z < 0.58) - P(Z < 0.35)$

$= 0.719 - 0.6368$

$= 0.0822 = 8.22\%$

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(c) $P(43.5 < X < 54.5)$

$= P(X < 54.5) - P(X < 43.5)$

$= P\left(Z < \frac{54.5-48}{4.3}\right) - P\left(Z < \frac{43.5-48}{4.3}\right)$

$= P(Z < 1.51) - P(Z < -1.05)$

$= 0.9345 - 0.1469$

$= 0.7876$

$= 78.76\%$

# of boxes accepted $= 130000 \times 0.7876$

$= 102388$ boxes

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