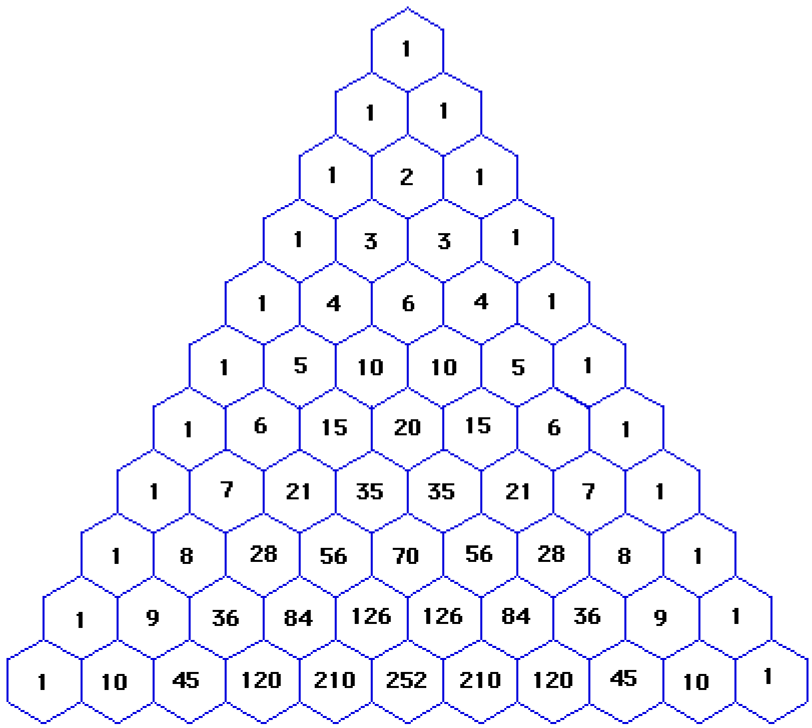
MDM4U **Pascal's Triangle** Date: ___________

Working with a partner, study the following array of numbers. What patterns do you see in the arrangement of the numbers? Describe each pattern using words and symbols.



As you look for patterns, try to answer the following questions:

1. Can you predict the next row of numbers?
2. Is there a pattern in the sums of the numbers in the rows?
3. Do any numbers repeat?
4. Can you find a pattern in the diagonal numbers?

See if you can find:

- ❖ natural numbers
  1, 2, 3, 4, ...
- ❖ powers of 2
  2, 4, 8, 16, ...
- ❖ powers of 11
  11, 121,1331,14641,
- ❖ triangular numbers
  1, 3, 6, 10, ...
- ❖ tetrahedral numbers
  1, 4, 10, 20, ...
- ❖ hexagonal numbers
  1, 6, 15, 28, ...
- ❖ Fibonacci numbers
  1, 1, 2, 3, 5, 8, ...
- ❖ Sierpinski triangle

The ***natural numbers*** are also known as the counting numbers. They appear in the second diagonal of Pascal's triangle:

1	Row 0
1 **1**	Row 1
1 **2** 1	Row 2
1 **3** **3** 1	Row 3
1 **4** 6 **4** 1	Row 4
1 **5** 10 10 **5** 1	Row 5
1 **6** 15 20 15 **6** 1	Row 6
1 **7** 21 35 35 21 **7** 1	Row 7
1 **8** 28 56 70 56 28 **8** 1	Row 8

Notice that the numbers in the rows of Pascal's triangle read the same left-to-right as right-to-left, so that the counting numbers appear in both the second left and the second right diagonal.

The ***powers of 2*** form a sequence:

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

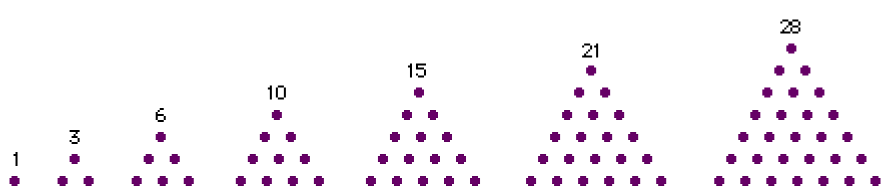
$2^7 = 128$

The ***sums of the rows*** in Pascal's triangle are equal to the powers of 2:

	Sum of Row	ROW
1	1	Row 0
1 1	2	Row 1
1 2 1	4	Row 2
1 3 3 1	8	Row 3
1 4 6 4 1	16	Row 4
1 5 10 10 5 1	32	Row 5
1 6 15 20 15 6 1	64	Row 6
1 7 21 35 35 21 7 1	128	Row 7
1 8 28 56 70 56 28 8 1		Row 8

A triangular number is a figurate number: a number that can be represented by a regular geometric arrangement of equally spaced points.

Triangular numbers can be thought of as the numbers of dots you need to make a triangle:

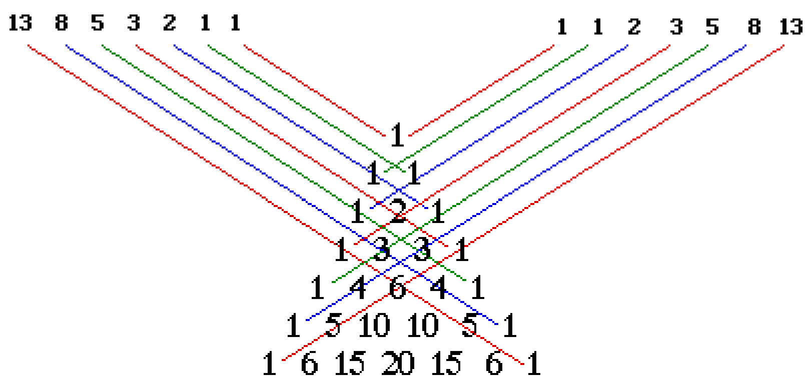


The triangular numbers are found in the third diagonal of Pascal's triangle:

1	Row 0
1 1	Row 1
1 2 **1**	Row 2
1 **3** **3** 1	Row 3
1 4 **6** 4 1	Row 4
1 5 **10** **10** 5 1	Row 5
1 6 **15** 20 **15** 6 1	Row 6
1 7 **21** 35 35 **21** 7 1	Row 7
1 8 **28** 56 70 56 **28** 8 1	Row 8

In the ***Fibonacci Sequence*** (1, 1, 2, 3, 5, 8, 13, ...), each term is the sum of the two previous terms (for instance, 2+3 = 5, 3+5 = 8, ...).

To find the Fibonacci numbers in Pascal's triangle you have to go up at an angle along the "shallow diagonals": you're looking for 1, 1, 1+1, 1+2, 1+3+1, 1+4+3, 1+5+6+1.



Each term in Pascal's triangle is equal to the sum of the two adjacent terms in the row immediately above it: In general we can denote $t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$ where $t_{n,r}$ represents the rth term in row n.

Discovering Patterns

Name ________________________