MDM4U **Permutations with Identical Items** Date: _____

☙ How many ways can we arrange the letters from MOO?

MO_1O_2
MO_2O_1

There are _____ distinct words.

☙ How many ways can we arrange the letters from MOOO?

$MO_1O_2O_3$
$MO_1O_3O_2$
$MO_2O_1O_3$
$MO_2O_3O_1$
$MO_3O_1O_2$
$MO_3O_2O_1$

There are only _____ different words.

☙ Now consider the letter MOOM

$M_1M_2O_1O_2$
$M_1M_2O_2O_1$
$M_2M_1O_1O_2$
$M_2M_1O_2O_1$

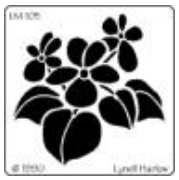
$\frac{4!}{2!\ 2!}$

In general, the permutations of n objects with "a" identical items, "b" identical items, "c" identical items and so on, the arrangement can be calculated using the formula

$$\frac{n!}{a!b!c!}$$

Example 1:

Determine the number of arrangements of 3 violets and 2 roses in a block vase.



n = a = b =

Example 2:

In how many ways could 5 pennies, 3 nickels, 2 dimes and a quarter be arranged in a line?

n = a = b = c = d =

Example 3:

a) How many arrangements are there of the letters in the word MATHEMATICS?

b) How many ways of these arrangements begin with the letter M?

c) How many of the arrangements in part a) would have the T's together?

Example 4:

How many 7-digit numbers can be formed using only the digits 1,1, 2,2,4,4,8?

Exercise 1:

In how many ways could 15 students be divided into 5 groups of 3?

Exercise 2:

Determine the number of arrangements of the letters in the following words:

a) TORONTO

b) MISSISSIPPI

c) CALCULUS

Exercise 3:

Jennifer is working on a word puzzle and is looking for four-letter "scrambles" from the clue word ***calculate***.

a) How many of the possible four-letter scrambles contain four different letters?

b) How many contain two a's and one other pair of identical letters?

c) How many scrambles consist of any two pairs of identical letters?

d) What possibilities have you not yet taken into account? Find the number of scrambles for each of these cases.

e) What is the total number of four-letter scrambles taking all cases into account?