MDM4U **Permutations with Identical Items** Date: _____

MO_1O_2 O_1MO_2 O_1O_2M
MO_2O_1 O_2MO_1 O_2O_1M
same same same

☙ How many ways can we arrange the letters from MOO?

MO_1O_2
MO_2O_1

$\frac{3!}{2!}$

There are 3 distinct words.

☙ How many ways can we arrange the letters from MOOO?

$MO_1O_2O_3$
$MO_1O_3O_2$
$MO_2O_1O_3$
$MO_2O_3O_1$
$MO_3O_1O_2$
$MO_3O_2O_1$

$\frac{4!}{3!}$

There are only 4 different words.

☙ Now consider the letter MOOM

$M_1M_2O_1O_2$
$M_1M_2O_2O_1$
$M_2M_1O_1O_2$
$M_2M_1O_2O_1$

2! 2!

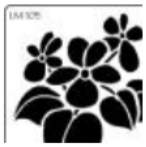
$\frac{4!}{2!2!} = 6$

In general, the permutations of n objects with "a" identical items, "b" identical items, "c" identical items and so on, the arrangement can be calculated using the formula $\frac{n!}{a!b!c!}$

MDM4U 2

Example 1:

Determine the number of arrangements of 3 violets and 2 roses in a block vase.



n = 5 a = 3 b = 2

$$\frac{5!}{3!\,2!} = 10$$

Example 2:

In how many ways could 5 pennies, 3 nickels, 2 dimes and a quarter be arranged in a line?

n = 11 a = 5 b = 3 c = 2 d = 1

$$\frac{11!}{5!\,3!\,2!} = 27720$$

Example 3:

a) How many arrangements are there of the letters in the word MATHEMATICS?

$$\frac{11!}{2!\,2!\,2!} = 4989600$$

b) How many ways of these arrangements begin with the letter M?

2 10 9 8 7 6 5 4 3 2 1

$$\frac{2(10!)}{2!\,2!\,2!} \text{ or } \frac{10!}{2!\,2!} = 907200$$

c) How many of the arrangements in part a) would have the T's together? Treat the 2T's as one choice

1 9 8 7 6 5 4 3 2 1

$$10 \times \frac{9!}{2!\,2!} = 907200$$

Example 4:

How many 7-digit numbers can be formed using only the digits 1,1, 2,2,4,4,8?

$\frac{7!}{2!2!2!} = 630$

5 groups $g_1 \to 3!$ $g_2 \to 3!$ $g_3 \to 3!$ $g_4 \to 3!$ $g_5 \to 3!$

Exercise 1:

In how many ways could 15 students be divided into 5 groups of 3?

$\frac{15!}{5!\,3!^5} = 1401400$

$g_1\ g_2\ g_3\ g_4\ g_5$

Exercise 2:

Determine the number of arrangements of the letters in the following words:

a) TORONTO

$\frac{7!}{2!3!} = 420$

b) MISSISSIPPI

$\frac{11!}{4!4!2!} = 34650$

c) CALCULUS

$\frac{8!}{2!2!2!} = 7! = 5040$

Exercise 3:

Jennifer is working on a word puzzle and is looking for four-letter "scrambles" form the clue word ***calculate***. 9 letters with 2 C's, 2 A's, and 2 L's plus U, T and E

a) How many of the possible four-letter scrambles contain four different letters?

6 5 4 3 $_6P_4$

$6 \times 5 \times 4 \times 3 = 360$

b) How many contain two a's and one other pair of identical letters?

AALL + AACC

$\frac{4!}{2!2!} + \frac{4!}{2!2!} = 12$

c) How many scrambles consist of any two pairs of identical letters?

AALL + AACC + CCLL

$\frac{4!}{2!2!} \times 3 = 18$

d) What possibilities have you not yet taken into account? Find the number of scrambles for each of these cases.

one pair of identical letters and two different letters

3 5 4 × 3 positions

= 180

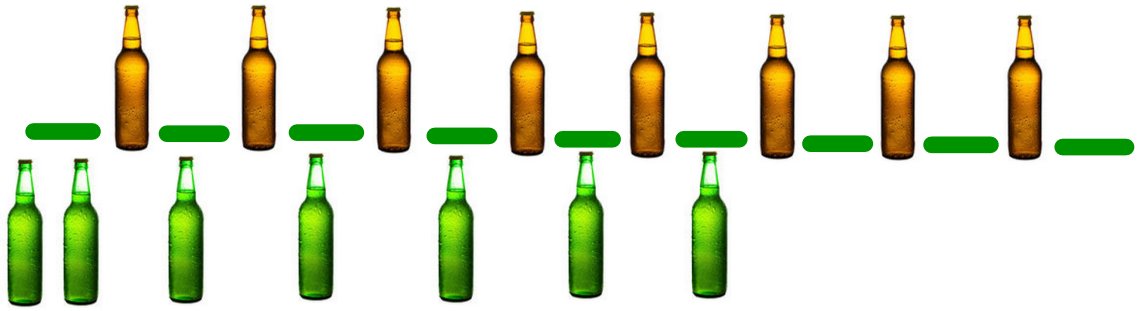
e) What is the total number of four-letter scrambles taking all cases into account?

558

Section 4.3 **Page 246** **Question 18**

18. How many different ways are there of arranging seven green and eight brown bottles in a row, so that exactly one pair of green bottles is side-by-side?

Place the 8 brown bottles in a row. There are now 9 places to put green bottles, 7 between the brown bottles and 1 at either end. Select 1 of the 9 spaces for the pair of green bottles. There are 9 ways of doing this. For each of these 9, you must select 5 of the remaining 8 spaces to hold one green bottle each. This is given by ${}_8P_5$. However, since the 5 green bottles are identical, you must divide by 5!. Therefore, the number of ways of placing the bottles so that exactly one pair of green bottles is together is given by $9 \times \frac{{}_8P_5}{5!} = 504$.



Brown × Green

$$\frac{8!}{8!} = 1 = \boxed{\frac{{}_8P_8}{8!}} \times \frac{{}_8P_5}{5!} \times 9$$