MDM4U Factorials and Permutations Date: ___________

1. How many ways can we arrange 10 books on a shelf? 

____ ____ ____ ____ ____ ____ ____ ____ ____ ____ = $10!$

This is a very long way to write a number. We will use a short notation for this operation from now. It is denoted by factorial.

For any natural number n, $n! = n(n-1)(n-2)(n-3)\ldots\ldots(3)(2)(1)$ Note: **0!=1** $50! = 50\times 49!$

Example 1: Working with factorials

$0! = 1$
$1! = 1$
$2! = 1\cdot 2 = 2$
$3! = 1\cdot 2\cdot 3 = 6$
$4! = 1\cdot 2\cdot 3\cdot 4 = 24$
$5! = 1\cdot 2\cdot 3\cdot 4\cdot 5 = 120$
$6! = 1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6 = 720$

$\frac{10!}{7!} =$

$\frac{100!}{98!}$

$6\times 5! =$

$(n+1)n! = (n+1)!$

$\frac{n!}{(n-2)!} =$

$\frac{(n+1)! + n!}{n!(n+1)!} = \frac{(n+1)n! + n!}{n!(n+1)!}$

$\frac{1}{n!}\frac{\times(n+1)}{\times(n+1)} + \frac{1}{(n+1)!} = \frac{\times n!\; n+1}{\times n!\;(n+1)!} + \frac{1}{(n+1)!} = \frac{n+2}{(n+1)!}$

$= \frac{n+1}{(n+1)n!}$

$= \frac{n+1}{(n+1)!}$

$\frac{n!(n+1+1)}{n!(n+1)!}$

$= \frac{\cancel{n!}(n+2)}{\cancel{n!}(n+1)!}$

$= \frac{n+1}{(n+1)!}$

$\frac{n!}{(n-k)!} = {}_nP_k$

$n(n-1)(n-2)\ldots(n-k+1)$

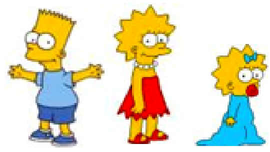
$\frac{10!}{6!} = 10\times 9\times 8\times 7$

A **permutation** of all elements of the set of size n is the number of **distinct** arrangements of the elements. It is denoted by ${}_nP_n = n!$ or $P(n,n)$.

Note: A permutation is an arrangement of elements whereby, if an element is selected, it cannot be selected again. In other words, no repeats is allowed

Example 2:

If the Simpsons (Bart, Lisa and Maggie) are to stand in a line for a photograph, how many arrangements could be made?



3!

For each of those 3 choices, there are 2 choices for the second position because the first person cannot be reused.

∴ There are 6 possible arrangements for these people.

Example 3:

Sandra has a blue, green, red, yellow and purple candy. In how many ways could they be lined up on a table?

5! = 120

A permutation of size ***r*** of ***n*** elements is the number of **distinct** arrangements of the ***r*** elements.

$${}_nP_r = \frac{n!}{(n-r)!}$$ Note: $n > r$ $P(n,r)$

nPr

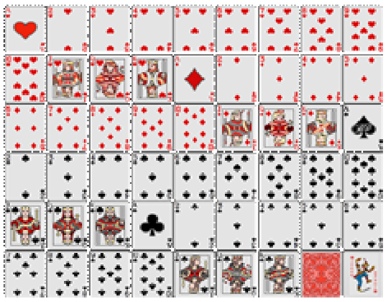
Example 4:

There are 10 magazines in a box. Five of them are to be placed onto a shelf in the library. In how many ways could they be arranged?

$_{10}P_5 = 30240$

Example 5:

From a standard deck of 52 cards, in how many ways could each of the following be arranged?



a) Five face cards (J,Q,K of 4 suits) 12 cards

4 4 4

$_{12}P_5 = 95040$

b) Eight hearts

$_{13}P_8 = 51891840$

c) Nine black cards

$_{26}P_9 = \frac{26!}{17!}$

$$\boxed{_nP_r = \frac{n!}{(n-r)!}}$$

Example 6:
In how many ways could the SAC, consisting of a president, vice president, treasurer and publicist be selected from 5 males and 5 females candidate if:

a) There are no restrictions? _ _ _ _

$_{10}P_4 = 5040$

b) The president and vice-president may not be of the same sex?

① $\underline{5}\ \underline{5}\ \underline{8}\ \underline{7}$

② $\times 2$

$5 \times 5 \times 8 \times 7 \times 2 = 2800$

Example 7:

Eric has a briefcase with a three-digit combination lock. He can set the combination himself, and his favourite digits are 5, 6, 7, 8 and 9. Each digit can be used at most once.

a) How many permutations of three of these five digits are there?

$${}_5P_3 = 5\times4\times3 = 60$$

b) If you think of each permutation as a three-digit number, how many of these numbers would be odd numbers?

$$\underline{4}\ \underline{3}\ \underline{3} = 36$$

c) How many of the three-digit numbers are even numbers and begin with a 8?

$$\underline{1}\ \underline{3}\ \underline{1} = 3$$

d) How many of the three-digit numbers are even numbers and do not begin with a 8?

Indirect method: ways for 3-digit even numbers - ways for 3-digit even numbers starting with 8

<u>Part (a) - Part (b) - Part (c) = Part (d)</u> = 21

If ends with 8 $\underline{4}\ \underline{3}\ \underline{1}$ If not end with 8 $\underline{3}\ \underline{3}\ \underline{1}$ $\boxed{12+9=21}$

e) Is there a connection among the four answers above? If so, state what it is and why it occurs.

They are connected by the indirect method because the total of 3-digit numbers is the sum of total 3-digit odd numbers and total 3-digit even numbers, and the total of 3 -digit even numbers is the sum of total 3-digit even number starting with 8 and total 3-digit even numbers not starting with 8.