Don't forget to write the answer statement for each question!

MDM4U **Probabilities Using Counting Techniques** Date: _____

We will look at examples of **probability** where the calculations involve ***permutation*** or ***combination*** formulas.

Example 1

Five boys and 7 girls have signed up for a ski trip. Only 4 will be chosen at random to go on the trip. Determine the probability that:

a) all will be boys

$$P(\text{all boys}) = \frac{{}_5C_4}{{}_{12}C_4} = \frac{1}{99}$$

b) all will be girls

$$P(\text{all girls}) = \frac{{}_7C_4}{{}_{12}C_4} = \frac{7}{99}$$

c) there will be 2 boys and 2 girls

$$P(\text{2 boys, 2 girls}) = \frac{{}_5C_2 \times {}_7C_2}{{}_{12}C_4} = \frac{14}{33}$$

Example 2

In a swim meet, there are 8 entries, 3 of whom come from the Halifax Swim Club. If we assume that their abilities are about the same, what is the probability that:

a) the Halifax swimmers, Anna, Ami and Amelia will finish first, second and third, respectively?

$$P(\text{Halifax in top three}) = \frac{1 \times 1 \times 1 \times {}_5P_5}{{}_8P_8} = \frac{120}{40320} = \frac{1}{336}$$

b) there will be no Halifax swimmers in the top three?

$$P(\text{no Halifax in top three}) = \frac{{}_5P_3 \times {}_5P_5}{{}_8P_8} = \frac{7200}{40320} = \frac{5}{28}$$

Example 3

In Lotto 6/49, 6 different numbers must be selected from the numbers 1 to 49. Calculate the odds against winning first prize.

$$P(1^{st}\text{ Prize}) = \frac{1}{{}_{49}C_6} = \frac{1}{13983816}$$

Odd against winning first prize = 13983816:1

Example 4

Three people form a queue at a grocery store. What is the probability that they line up in descending order of age?

$$P(\text{descending order}) = \frac{1}{{}_3P_3} = \frac{1}{6}$$

Example 5

Nine horses are entered in a race. In an attempt to predict the finish of the race, three horses are selected by lot to finish first, second and third. What is the probability that the choice is correct?

$$P(\text{correct choice order}) = \frac{1}{{}_9P_3} = \frac{1}{504}$$

Example 6

A committee of five people is to be selected from ten males and eight females. What is the probability that there are exactly three males on the committee?

P(exactly 3 males) = $\frac{{}_{10}C_3 \times {}_8C_2}{{}_{18}C_5} = \frac{3360}{8568} = \frac{20}{51}$

Example 7

The school yearbook is to be produced by a student staff of two boys and three girls, chosen by lot from five boys and six girls. One of the boys is the boyfriend of one of the girls. What is the probability that both will be chosen to be on the editorial staff of the yearbook?

P(both chosen) = $\frac{{}_2C_2 \times {}_4C_1 \times {}_5C_2}{{}_5C_2 \times {}_6C_3} = \frac{40}{200} = \frac{1}{5}$

This is newly added. Don't forget that we can always calculate probability using the basic counting principle.

Example 8

A man can hit a target once in 4 shots. If he fires 4 shots in succession, what is the probability that he will hit his target at least once?

P(hit the target) = $\frac{1}{4}$ P(miss the target) = $\frac{3}{4}$

P(hit at least once with 4 successive shots) = 1 – P(miss all 4 shots)

$= 1 - \left(\frac{3}{4}\right)^4$

$= 1 - \frac{81}{256}$

$= \frac{175}{256}$

Example 9

A telephone company is assigned a new area code and can issue new 7-digit phone numbers. All phone numbers are equally likely.

(a) Find the probability of being assigned the phone number 333-6789.

$P(333\text{-}6789) = \frac{1}{10^7}$

$= \frac{1}{10000000}$

(b) Find the probability of a phone number that does not contain a 5.

$P(\text{no } 5) = \frac{9^7}{10^7}$

$= \frac{4782969}{10000000}$